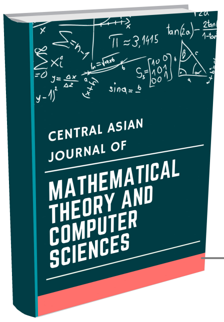

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Solving Non-Line Parabolic Equations

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Abstract:

This paper discusses the problems of numerical modeling of the Cole-Hopf equation by the spectral method. An algorithm for the spectral method is developed for numerical modeling of the problem, which is applied to the numerical modeling of the Cole-Hopf equation.

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INTRODUCTION

Numerous scientific and practical researches are carried out in the world, mostly on the modeling of nonlinear wave processes, numerical modeling of wave propagation problems and construction of approximate solutions, increasing the accuracy of approximate solutions. Modeling of nonlinear processes and the development of methods for their numerical calculation are the object of research in the fields of gas dynamics, hydrodynamics, acoustics, optics, plasma physics, radio physics, electronics. Therefore, the construction of nonlinear mathematical models covering natural processes, the construction of efficient numerical solution schemes and algorithms, and the creation of their software are important in the field of applied mathematics.

The article [1] describes the numerical modeling of special derivative differential equations by the spectral method. In the article [2] the problems of hydrodynamic stability are approximated by the spectral-grid method and its special spectral method. In the articles [3-8] the authors consider the problems of mathematical modeling of linear and nonlinear evolutionary equations. The authors considered the modeling of Navier-Stokes equations of direct and inverse problems of vixr-tok functions by the method of differential schemes [9-10]. Articles [11-13] describe methods for solving simple differential equations.

Main part.

The following initial-boundary value problem is considered for the Cole-Hopf equation:

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x}, a < x < b, \tag{1}$$

$$\begin{aligned} u(a,t) &= 0, \\ u(b,t) &= 0, \end{aligned} \tag{2}$$

$$u(x,0) = u_0(x). \tag{3}$$

We use the spectral method for numerical modeling of the differential problem (1) - (3). To do this, the approximate solution of the differential problem (1) - (3) is sought in the form of a series of Chebishev polynomials of the first type:

$$u(y)=\sum_{n=0}^{N} a_n T_n(y),\ \ T_n(y)=\cos(n\cdot \arccos y), \tag{4}$$

where, $T_n(y)$ is the Chebyshev polynomial and N is the number of polynomials used to approximate the approximate solution in $[a,b]$ sections. Section $[a,b]$ has $(N+1)$ collocation points, which consist of nodes of the Chebishev polynomial:

$$y_l=\cos(\pi\, l/N),\ \ l=0,1,\ldots,N\,.$$

The coefficients a_m in series (4) are determined by the values of function $u(y_l)$ by this inverse Chebishev substitution:

$$\begin{gathered} a_m=\frac{2}{Nc_m}\sum_{l=0}^{N}\frac{1}{c_l}u(y_l)T_m(y_l), \quad m=0,1,\ldots,N, \\ c_0=c_N=2,\ c_m=1, \qquad if\ m\neq 0,N. \end{gathered} \tag{5}$$

Rows (4) and (5) are written in the form of the following matrix:

$$v=Ta\,, \tag{6}$$

$$a=T^{*}v\,, \tag{7}$$

where, $a=\{a_0,a_1,a_2,\ldots,a_N\}$ is a vector of coefficients, T and T^* are square matrices with dimensions $(N+1)\times(N+1)$:

$$T=\begin{bmatrix} T_0(y_0) & T_1(y_0) & \cdots & T_{N-1}(y_0) & T_N(y_0) \\ T_0(y_1) & T_1(y_1) & \cdots & T_{N-1}(y_1) & T_N(y_1) \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ T_0(y_{N-1}) & T_1(y_{N-1}) & \cdots & T_{N-1}(y_{N-1}) & T_N(y_{N-1}) \\ T_0(y_N) & T_1(y_N) & \cdots & T_{N-1}(y_N) & T_N(y_N) \end{bmatrix},$$

$$T^{*}=\begin{bmatrix} \frac{T_0(y_0)}{4} & \frac{T_0(y_1)}{2} & \cdots & \frac{T_0(y_{N-1})}{2} & \frac{T_0(y_N)}{4} \\ \frac{T_1(y_0)}{2} & T_1(y_1) & \cdots & T_1(y_{N-11}) & \frac{T_1(y_N)}{2} \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ \frac{T_{N-1}(y_0)}{2} & T_{N-1}(y_1) & \cdots & T_{N-1}(y_{N-1}) & \frac{T_{N-1}(y_N)}{2} \\ \frac{T_N(y_0)}{4} & \frac{T_N(y_1)}{2} & \cdots & \frac{T_N(y_{N-1})}{2} & \frac{T_N(y_N)}{4} \end{bmatrix},$$

and v vector and its components are as follows:

$$v \equiv \{u(y_0), u(y_1), u(y_2), \ldots, u(y_N)\}. \tag{8}$$

Similarly, in y_l discrete collocation nodes, the first spatial product can be expressed as follows:

$$\frac{\partial v}{\partial y} = Tb, \tag{9}$$

where b vector components are defined by a vector components as follows:

$$c_m b_m = 2 \sum_{\substack{p=m+1 \\ p \equiv m (\mathrm{mod}\, 2)}}^{N} p a_p, \; m \geq 0, \tag{10}$$

where, setting $r \equiv q(\mathrm{mod} 2)$ means that expression $(r-q)/2$ takes the whole value.

Formula (10) is written in matrix form as follows

$$b = Ra, \tag{11}$$

where R is a square matrix of size $(N+1)\times(N+1)$:

$$R = \begin{bmatrix} 0 & 1 & 0 & 3 & 0 & \mathrm{K} & x & x \\ 0 & 0 & 4 & 0 & 8 & \mathrm{K} & x & x \\ 0 & 0 & 0 & 6 & 0 & \mathrm{K} & x & x \\ 0 & 0 & 0 & 0 & 8 & \mathrm{K} & x & x \\ 0 & 0 & 0 & 0 & 0 & \mathrm{K} & x & x \\ \mathrm{M} & \mathrm{M} & \mathrm{M} & \mathrm{M} & \mathrm{M} & \mathrm{M} & \mathrm{M} & \mathrm{M} \\ 0 & 0 & 0 & 0 & 0 & \mathrm{K} & 0 & x \\ 0 & 0 & 0 & 0 & 0 & \mathrm{K} & 0 & 0 \end{bmatrix}$$

Substituting formula (11) into (9) and taking into account the relation (7), the approximation of the spatial product is obtained:

$$\frac{\partial v}{\partial y} = \hat{B} v \tag{12}$$

Matrix $\hat{B}$ is a square matrix of size $(N+1)\times(N+1)$, which is calculated in the following order:

$$\hat{B} = TRT^{*}. \tag{13}$$

Auxiliary $\widetilde{B}$ matrix is also included:

$$\widetilde{B} = \hat{B}\,. \tag{14}$$

By writing the differential equation (1) at $(l = 1,\ldots,N-1)$ internal collocation points, and the conditions (2) at the boundary points of the interval, we get the following system:

$$\frac{dS}{dt} = -v \cdot Bv\,, \tag{15}$$

where the full stop represents the product of the components of the two vectors, denoted by S a vector of length $N+1$:

$$S \equiv \{0, u(y_1), u(y_2), \ldots u(y_{N-1}), 0\},$$

B matrix has $(N+1)\times(N+1)$ dimensions and looks like this:

$$B = \begin{bmatrix} 0 & 0 & \Lambda & 0 & 0 \\ b_{10} & b_{11} & \Lambda & b_{1N-1} & b_{1N} \\ M & M & \Lambda & M & M \\ b_{N-10} & b_{N-11} & \Lambda & b_{N-1N-1} & b_{N-1N} \\ 0 & 0 & \Lambda & 0 & 0 \end{bmatrix}$$

where, $b_{ij}, (i \neq 0, N)$ elements of B matrix are equal to $\widetilde{b}_{ij}, (i \neq 0, N)$ corresponding elements of $\widetilde{B}$ matrices.

Thus, Equation (15), which consists of a system of simple differential equations, can be solved by using a combination of one-step Runge-Kutta and multi-step Adams-Beshfort methods.

Results and discussion.

The developed algorithm was applied to the numerical modeling of the initial-boundary conditional problem for the Cole-Hopf equation. Thus the following problem is considered for the Cole-Hopf equation:

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x}, \tag{16}$$

$$\begin{aligned} u(x,0) &= -\sin \pi x, \\ u(0,t) &= 0, u(1,t) = 0. \end{aligned} \tag{17}$$

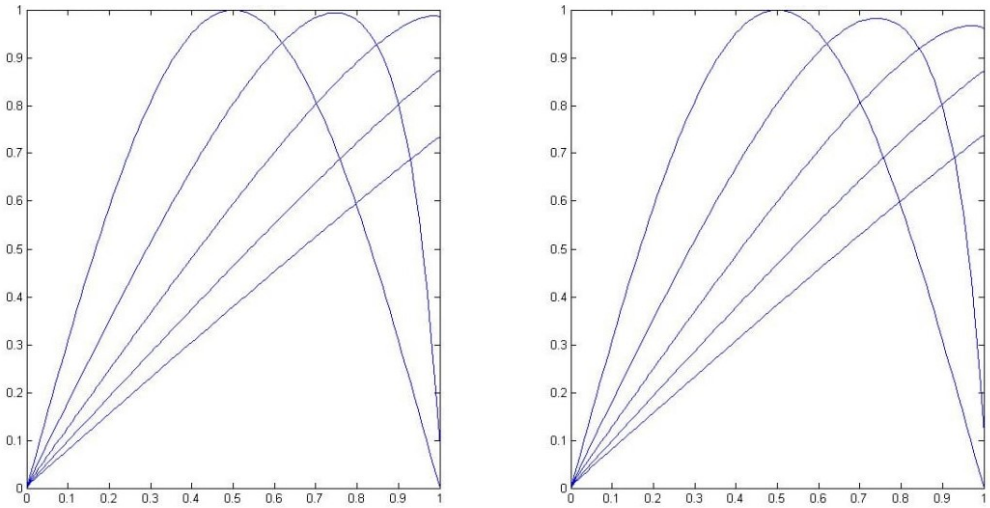

Figure 1. The solution obtained by the differential scheme method.

The computational experiment using the method of differential schemes was carried out at the following values of the parameters: $N = 100, \tau = 10^{-2} / 6\pi$. The results from the graphs seem very good, but in the trajectory of the shock wave, the front of the wave moves forward, and when the moment of time is $t > 0.4$, the phenomenon of rolling of the wave occurs. In this graph, when the time moment is $t > 0.4$, the wave appears as if it had been cut off, which leads to a calculation error.

The computational experiment using the spectral method was performed at the following values of the parameters: $n = 32, \tau = 10^{-2} / 6\pi$. As can be seen from Figure 2, as the value of the first-order product increases at point $x = 0$, oscillations begin in the numerical solution. When the moment of time is $t = 0.425$, due to the increase in the amplitude of oscillations, intermittent oscillations occur at all points in the field under consideration. When the time moment is $t > 0.45$, the amplitude of the oscillations becomes so large that not a single exact number remains in the numerical solution.

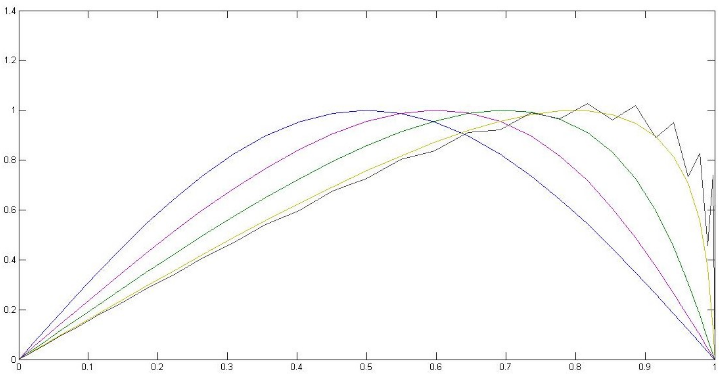

Figure 2. Spectral method.

It is possible to increase the number of nodes to eliminate errors in the form of intermittent oscillations of the solution. This method can delay the appearance of intermittent waves, but does not eliminate them. Excessively increasing the number of nodes requires the computer to perform redundant operations.

Conclusion

Numerical modeling of nonlinear parabolic equations requires serious difficulties. The most important of them is the presence of a thin layer in the area under consideration of the solution, in which there is a sharp change in the solution and the solution assumes a state of interruption. In this case, the demand for the approximation of numerical methods increases sharply. Therefore, although there are a number of methods for solving these types of equations, the question of their effectiveness, accuracy and the extent to which they reflect the nature of the solution in the above-mentioned thin layer remains relevant.

References

1. Игумнов А.Б., Соловьев А.С. Устойчивый псевдоспектральный метод решения начально–краевых задач. // Ж. вычисл. матем. и матем. физ. – 1986. – Т. 26. – № 12. – С. 1895-1900.
2. Нармурадов Ч. Б. Математическое моделирование гидродинамических задач для двухфазных плоскопараллельных течений //Математическое моделирование. – 2007. – Т. 19. – №. 6. – С. 53-60.
3. Normurodov C. B., Toyirov A. X., Yuldashev S. M. Numerical modeling of nonlinear wave systems by the spectral-grid method //International Scientific Journal Theoretical & Applied Science, Philadelphia, USA. – 2020. – Т. 83. – №. 3. – С. 43-54.
4. Narmuradov C. B. et al. MATHEMATICAL MODELING OF MOVEMENT OF A VISCOUS INCOMPRESSIBLE LIQUID BY THE SPECTRAL-GRID METHOD //Theoretical & Applied Science. – 2020. – №. 4. – С. 252-260.
5. Begaliyevich N. C., Khasanovich T. A. Spectral-grid method for solving evolution problems with high gradients //EPRA International Journal of Multidisciplinary Research (IJMR). – Т. 67.
6. Нармурадов Ч. Б., Тойиров А. Х. Математическое моделирование нелинейных волновых систем //Проблемы вычислительной и прикладной математики. – 2018. – №. 1. – С. 21-31.
7. BEGALIYEVICH N. C. et al. Mathematical Modeling of the Hydrodynamic Stability Problem by the Spectral-grid Method //International Journal of Innovations in Engineering Research and Technology. – Т. 7. – №. 11. – С. 20-26.
8. Toyirov A. K., Yuldashev S. M., Abdullayev B. P. Numerical modeling the equations of heat conductivity and burgers by the spectral-grid method //НАУКА 2020. ТЕОРИЯ И ПРАКТИКА. – 2020. – С. 30-31.
9. Нармурадов Ч. Б., Гуломкодиров К. А. Математическое моделирование уравнений Навье-Стокса в системе вихря и функции тока //Проблемы вычислительной и прикладной математики. – 2017. – №. 3. – С. 29-32.
10. Нармурадов Ч. Б., Холияров Э. Ч., Гуломкодиров К. А. Численное моделирование обратной задачи релаксационной фильтрации однородной жидкости в пористой среде //Проблемы вычислительной и прикладной математики. – 2017. – №. 2. – С. 12-19.
11. Abdirasulovna Z. S., Majidovna N. M. Evaluation of Errors in Numerical Solution of Problems //CENTRAL ASIAN JOURNAL OF MATHEMATICAL THEORY AND COMPUTER SCIENCES. – 2021. – Т. 2. – №. 9. – С. 45-47.
12. Abdirasulovna Z. S. Conducting a Computational Experiment using Test Functions //CENTRAL ASIAN JOURNAL OF MATHEMATICAL THEORY AND COMPUTER SCIENCES. – 2021. – Т. 2. – №. 9. – С. 51-53.
13. Shavkatovna D. Z. Solving Cauchy Problems Using Euler Methods Using the C# Programming Language and Method Mapping //International Journal of Innovative Analyses and Emerging Technology. – 2021. – Т. 1. – №. 4. – С. 74-77.