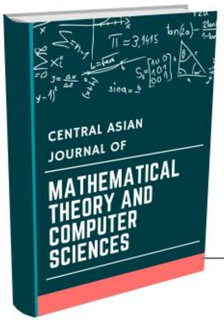

CENTRAL ASIAN JOURNAL OF MATHEMATICAL THEORY AND COMPUTER SCIENCES

http://cajmtcs.centralasianstudies.org/index.php/CAJMTCS

Volume: 02 Issue: 12 | Dec 2021 ISSN: 2660-5309

Numerical Modeling of Heat Conduction Equation with Piecemeal Intermittent Continuous Coefficient

Normurodov Chori Begaliyevich

Professor, Department of Applied mathematics and informatics, Termez state university

Yuldashev Shamsiddin Mamarajabovich

(PhD) doktorant, Department of Applied mathematics and informatics, Termez state university

Abstract:

In this paper, the thermal conductivity equation with a continuous coefficient of fractional continuity is numerically modeled by differential methods and spectral-grid methods. For numerical modeling of the problem, a numerical solution algorithm was developed on the basis of differential schemes and spectral-grid methods, software for the algorithm was developed, and the numerical results were analyzed. Extensive computational experiments were performed at different values of the characteristic parameters, the numerical results obtained and their analysis were presented.

ARTICLEINFO

Article history:
Received 30 Oct 2021
Revised form 22 Nov 2021
Accepted 27 Dec 2021

Keywords: numerical modeling, spectral method, differential schemes, evolutionary problems, Chebyshev polynomials of the first kind, approximate solution.

---***---

Introduction: Numerous scientific and practical studies are conducted around the world, mostly on the modeling of quasi-linear temperature processes, numerical modeling of heat dissipation problems and construction of approximate solutions, increasing the accuracy of approximate solutions. Modeling of quasi-linear processes and the development of methods for their numerical calculation are the subject of research in the fields of gas dynamics, hydrodynamics, acoustics, optics, plasma physics, radiophysics, electronics. Therefore, the construction of nonlinear mathematical models involving natural processes, the construction of efficient numerical solution schemes and algorithms, and the creation of their software are important in the field of applied mathematics.

The article [1] describes the numerical modeling of special derivative differential equations by the spectral method. In the article [2] the problems of hydrodynamic stability are approximated by the spectral-grid method and its special spectral method. The article [3] considers the calculation of the quasi-linear thermal conductivity equation using differential schemes. In the article [4], the equation of thermal conductivity, which is a small parameter in the presence of a high-order product, is numerically modeled by the spectral-grid method. In the articles [5-10] the authors consider the problems of mathematical modeling of linear and nonlinear evolutionary equations. The authors considered the modeling of Navier-Stokes equations of direct and inverse problems of vixr-tok functions by the method of differential schemes [11-12]. The article [13] is devoted to finding the absolute and relative errors of solutions of equations. The author describes an article on the use of test functions in solving differential equations [14]. Articles [15-16] describe methods for solving simple differential equations.

Main part.

Given the following initial-boundary value problem for the equation of thermal conductivity with a constant coefficient of fractional continuity:

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(k(x)\frac{\partial u}{\partial x}\right), a < x < b,$$

$$u(a,t) = 0,$$

$$u(b,t) = 0,$$

$$u(x,0) = u_0(x).$$

The differential equation is given by:

$$\frac{\partial u}{\partial t} = \frac{\partial k(x)}{\partial x} \cdot \left(\frac{\partial u}{\partial x}\right) + k(x)\frac{\partial^2 u}{\partial x^2}.$$

In this equation, let *k(x)* be $k_i, i \geq 4$ numbers that do not change in the given area.

The following initial boundary issues are considered:

$$\frac{\partial u}{\partial t} = k_i \frac{\partial^2 u}{\partial x^2}, a < x < b, \; i \geq 4 \qquad (1)$$

$$u(a,t) = 0, \quad u(b,t) = 0, \qquad (2)$$

$$u(x,0) = u_0(x). \qquad (3)$$

Differential method. The numerical solution of the initial-boundary value problem (1) - (3) is considered by the method of finite difference.

$$D = \{a \leq x \leq b, \; 0 \leq t \leq T\}$$

The difference grid in the field is entered as follows:

$$\overline{\omega}_{h\tau} = \left\{(x_i, t_j), \begin{array}{l} x_i = ih, i = 0,1,2,..., N, \quad h = (b-a)/N, \\ t_j = j\tau, j = 0,1,2,...,M, \tau = T/M \end{array}\right\}.$$

The problem (1) - (3) is approximated using a purely transparent differential scheme. As a result, the differential problem (1) - (3) leads to the difference problem in the following form:

$$\frac{y_i^{j+1} - y_i^j}{\tau} = \frac{1}{h}\left[\frac{k_{i+1} + k_i}{2}\frac{y_{i+1}^{j+1} - y_i^{j+1}}{h} - \frac{k_i + k_{i-1}}{2}\frac{y_i^{j+1} - y_{i-1}^{j+1}}{h}\right], \begin{array}{l} i = 1,2,...N-1, \\ j = 0,1,...M-1, \end{array} \qquad (4)$$

$$y_0^{j+1} = 0, \; y_N^{j+1} = 0, \; j = 0,1,...M-1, \qquad (5)$$

$$y_i^0 = u_0(x_i), \; i = 0,1,...N. \qquad (6)$$

To use the prognosis method to solve the difference problem (4) - (6), Equation (4) is written as follows:

$$Ay_{i-1}^{j+1} - Cy_i^{j+1} + By_{i+1}^{j+1} = -F_i, \qquad (7)$$

where

$$A=\frac{(k_i+k_{i-1})\tau}{2h^2},\ B=\frac{(k_{i+1}+k_i)\tau}{2h^2},\ C=1+\frac{((k_{i+1}+k_i)+(k_i+k_{i-1}))\tau}{2h^2},\ F_i=y_i^j;$$

the differential equation (7) is solved numerically using the prognosis method with conditions (5) - (6). In this case, the prognosis coefficients are as follows:

$$y_i^{j+1}=\alpha_{i+1}y_{i+1}^{j+1}+\beta_{i+1},\ i=N-1,\dots 1,0,\ j=0,1,\dots,M-1,$$
$$y_N^{j+1}=0, \qquad (8)$$

$$\alpha_{i+1}=\frac{B}{C-A\alpha_i},\ \beta_{i+1}=\frac{A\beta_i+F_i^j}{C-A\alpha_i},\ i=1,2,\dots,N-1, \qquad (9)$$

To find the initial prognosis coefficients, put $i=0$ in (8) and divide by:

$$y_0^{j+1}=\alpha_1 y_1^{j+1}+\beta_1. \qquad (10)$$

Comparing Equation (10) with the boundary condition (5), the following expression is formed [3]:

$$y_0^{j+1}=\alpha_1 y_1^{j+1}+\beta_1=0,\ \alpha_1=0,\ \beta_1=0. \qquad (11)$$

expression (9) is the correct path of the prognosis method, and expression (8) is the inverse of the progonka method.

Spectral-grid method. In the field under consideration, a grid is included in the integration interval $[a,b]$, which is applied to the spectral-grid method [4] to solve the differential problem (1) - (3) set for the thermal conductivity equation with a constant coefficient of fragmentation. that is, this section is divided into M different elements:

$$[x_0,x_1],[x_1,x_2],[x_2,x_3],\dots,[x_{i-1},x_i],\dots,[x_{M-1},x_M],$$

in this case $x_0=a$, $x_M=b$. To express the approximate solution of the differential problem (1) - (3) in the form of a series of Chebyshev polynomials of the first type, the formula for substituting the following variable is used to reflect each of the $[x_{j-1},x_j]$ elements of the considered $[a,b]$ intersections into $[-1,1]$ intersections.

$$x_j=\frac{m_j}{2}+\frac{l_j}{2}y, \qquad (12)$$

where $m_j=x_j+x_{j-1}$, $l_j=x_j-x_{j-1}$ is the length of element j of the grid and $y\in[-1,1]$. The differential problem (1) - (3) considered in each element of the entered network has the following form:

$$\frac{\partial u_i}{\partial t}=k_j\left(\frac{2}{l_i}\right)^2\frac{\partial^2 u_i}{\partial y^2},\ i=1,2,\dots,M,\ \ j\geq 2 \qquad (13)$$

$$u_i(1)=u_{i+1}(-1),\ \ i=1,2,\dots,M-1, \qquad (14)$$

$$\frac{1}{l_i}\frac{\partial u_i}{\partial y}(1) = \frac{1}{l_{i+1}}\frac{\partial u_{i+1}}{\partial y}(-1), \quad i = 1,2,\ldots,M-1, \qquad (15)$$

$$u_1(-1) = u_M(1) = 0, \qquad (16)$$

$$u_i(x,0) = u_0\left(\frac{m_i}{2} + \frac{l_i}{2}y, 0\right), \quad i = 1,2,\ldots,M, \qquad (17)$$

where equations (14) - (15) determine the approximate solution and the condition of continuity of its first-order product within the boundaries of adjacent grid elements, equation (16) defines the boundary conditions, and equation (17) defines the initial conditions.

The approximate solution of the system of equations (14) - (16) is used Chebishev polynomials of the first type, second-order spatial products at y_i discrete collocation nodes, and linear nonlinear transformations [5-7]. As a result, equation (14) looks like this:

$$\frac{dr}{dt} = Hr, \qquad (18)$$

The solutions of the system of equations (18) can be found using single-step Runge-Kutta and multi-step Adams-Beshfort schemes [8-10].

Results and discussion.

The algorithm developed above using differential schemes and spectral-grid methods has been applied to the calculation of the initial-boundary value problem for the thermal conductivity equation (1) - (3) with a continuous variable coefficient.

In this case, the initial solution to the problem is considered in this way [4].

$$u(x,0) = \exp\left[-\frac{k_i x^2}{4t_0}\right], i \geq 2 \qquad (19)$$

The constant t_0 determines the half-length of the initial distribution: t_0 the smaller it is, the shorter it will be. Numerical calculations were performed mainly at the following values of the parameters: $t_0 = 0.15$, $\tau = 0.01$. The integration interval is selected to be $[-1,1]$. For a given half-length of the initial distribution, the function (19) is zero with 10^{-12} accuracy at the boundary points [11-12]. Therefore, equation (19) is considered with the following boundary conditions:

$$u(\pm 1, t) = 0, \qquad (20)$$

The computational experiment was performed using a differential scheme with $N = 16$, $N = 32$, $N = 64$ and $N = 96$ nodes when the $k_1 = 150$, $k_2 = 200$, $k_3 = 250$, $k_4 = 300$ and $[-1,1]$ sections were divided into equal intervals. Similarly, the basis functions (Chebishev polynomials) were performed by the spectral-grid method with the number $N = 16$, $N = 32$ and $N = 64$ in $t = 40\tau$ layers of time, where the grid step is τ times.

Table 1. Differential method, in this $k_1 = 150, k_2 = 200, k_3 = 250, k_4 = 300$

	-0.1	-0.05	0	0.05	0.1
N=16	0.2429	0.5201	0.7972	0.5155	0.2336
N=32	0.2065	0.3934	0.5748	0.3666	0.1747
N=64	0.2060	0.4105	0.5314	0.3902	0.1654
N=96	0.2072	0.4140	0.5276	0.3923	0.1655

Table 2. Spectral-grid method, in this $k_1 = 150, k_2 = 200, k_3 = 250, k_4 = 300$

	-0.1	-0.05	0	0.05	0.1
N=16	0.2539	0.4279	0.5261	0.4065	0.2488
N=32	0.2412	0.378	0.4672	0.3623	0.2224
N=64	0.2257	0.3736	0.4474	0.3583	0.2074
N=96	0.2239	0.3769	0.4482	0.361	0.2045

The thermal conductivity of the grid elements k_i is now selected as follows: Computational experiments are performed when the $k_1 = 200,\ k_2 = 300,\ \ k_3 = 350,\ k_4 = 250$ and $[-1,1]$ sections are divided into equal intervals.

Table 3. Differential method, in this $k_1 = 200,\ k_2 = 300,\ k_3 = 350,\ k_4 = 250$

	-0.1	-0.05	0	0.05	0.1
N=16	0.2286	0.5416	0.8544	0.5398	0.2251
N=32	0.1408	0.3378	0.6223	0.3170	0.1223
N=64	0.1311	0. 3635	0.5367	0.3455	0.1063
N=96	0.1316	0.3681	0.5297	0.3487	0.1057

Table 4. Spectral-grid method, in this $k_1 = 200,\ k_2 = 300,\ k_3 = 350,\ k_4 = 250$

	-0.1	-0.05	0	0.05	0.1
N=16	0.2033	0.379	0.4773	0.3483	0.1935
N=32	0.1929	0.3397	0.4571	0.3312	0.1834
N=64	0.1726	0.3379	0.4338	0.3274	0.1616
N=96	0.1663	0.3391	0.4332	0.3279	0.1549

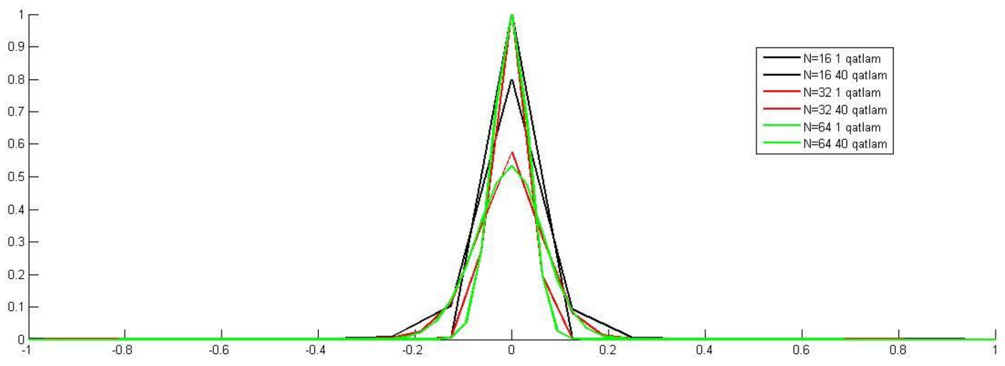

Figure 1. A solution calculated by different methods in different grids and at different time layers.

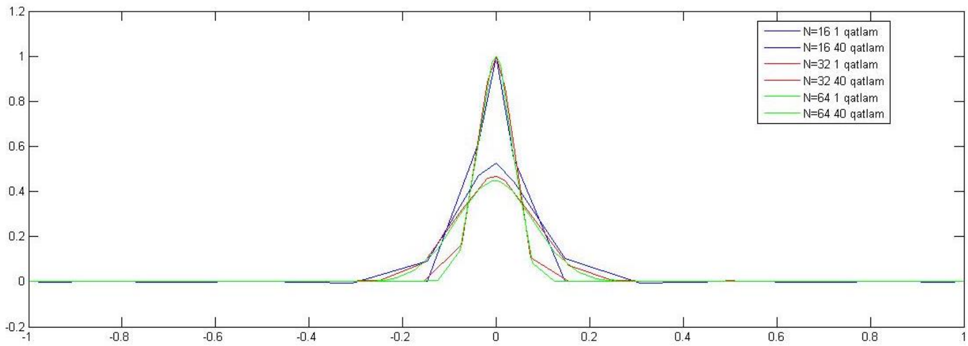

Figure 2. Solution by the spectral-grid method for obtaining different numbers of Chebyshev polynomials and calculating them in different time layers.

At the center of section [-1,1], the coefficient of thermal conductivity k_i is sharply variable, ie distributed as follows: [-1,-0.2] in section $k_1 = 200,$ [-0.2,0] in section $k_2 = 300,$ [0 , 0.2] section $k_3 = 350,$ [0.2,1] section $k_4 = 250.$

Table 5. Differential method, in this $k_1 = 200, k_2 = 300, k_3 = 350, k_4 = 250$

	-0.1	-0.05	0	0.05	0.1
N=16	0.2282	0.5413	0.8544	0.5395	0.2245
N=32	0.1408	0.3378	0.6223	0.3170	0.1222
N=64	0.1312	0.3635	0.5367	0.3455	0.1062
N=96	0.1315	0.3681	0.5297	0.3487	0.1057

Table 6. Spectral-grid method, in this $k_1 = 200, k_2 = 300, k_3 = 350, k_4 = 250$

	-0.1	-0.05	0	0.05	0.1
N=16	0.2033	0.379	0.4773	0.3483	0.1935
N=32	0.1929	0.3397	0.4571	0.3312	0.1834
N=64	0.1726	0.3379	0.4338	0.3274	0.1616
N=96	0.1663	0.3391	0.4332	0.3279	0.1549

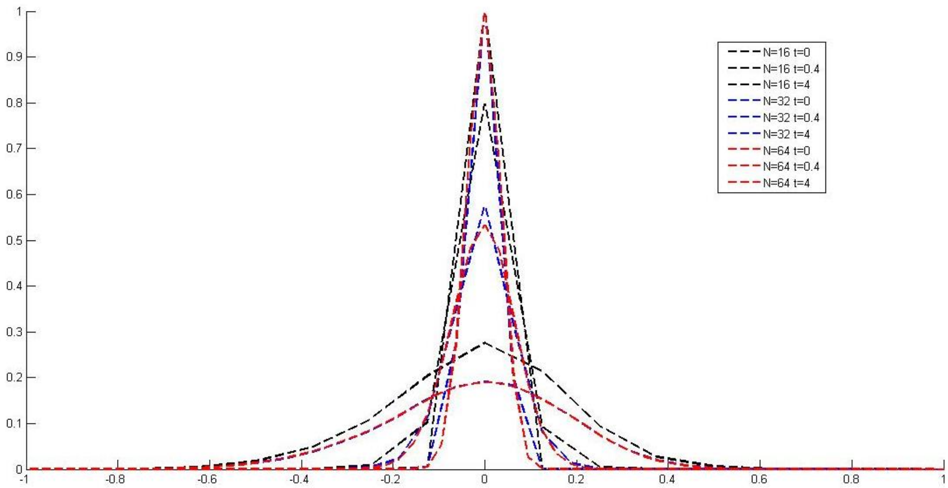

Figure 3. Solution obtained by different methods in different networks and time layers.

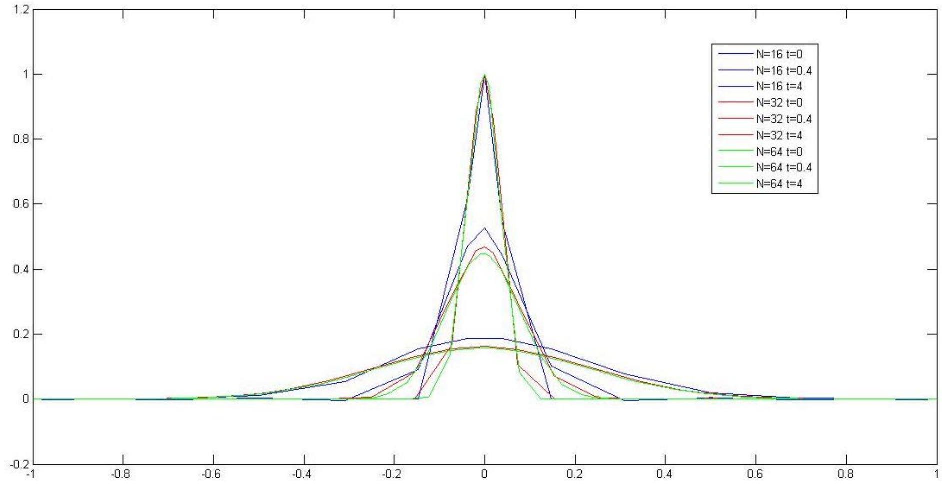

Figure 4. A solution calculated by the spectral-grid method when different numbers of Chebyshev polynomials are obtained and calculated in different time layers.

Tables 1, 3, 5, and Figures 1 and 3 show a comparison of the approximate solutions of the problem under different methods with the different values of parameter k_i obtained when the number of nodes in the network is $N=16$, $N=32$, $N=64$ and $N=96$. It can be seen that the values of the solutions obtained at different values of the selected k_i parameters, as well as at different lattice and time layers, differ significantly [13-14].

At present, in Tables 2, 4, 6 and Figures 2, 4, the results of the comparison of the numerical solutions obtained by the spectral-grid method of the problem under consideration at different values of parameter k_i show that the number of Chebishev polynomials is $N=16$, $N=32$, $N=64$ and $N=96$, the approximate solutions of the problem obtained by the spectral-grid method are almost identical in the selected values of k_i parameters, which in turn showed that they can have high accuracy using a small number of polynomials [15-16].

Conclusion

The results show that the spectral-grid method is universal, with the possibility of choosing the number of grid elements of different lengths and the number of polynomials used to approximate the solution. As a result, the solution is found with high accuracy.

References

1. Игумнов А.Б., Соловьев А.С. Устойчивый псевдоспектральный метод решения начально–краевых задач. // Ж. вычисл. матем. и матем. физ. – 1986. – Т. 26. – № 12. – С. 1895-1900.
2. Нармурадов Ч. Б. Математическое моделирование гидродинамических задач для двухфазных плоскопараллельных течений //Математическое моделирование. – 2007. – Т. 19. – №. 6. – С. 53-60.
3. Нармурадов Ч. Б., Тойиров А. Х., Юлдашев Ш. М. Численное моделирование параболических уравнений с малым параметром при старшей производной //Проблемы вычислительной и прикладной математики. – 2020. – №. 6. – С. 68-78.
4. Нармурадов Ч.Б., Юлдашев Ш.М., Тойиров А.Х. Математическое моделирование нелинейных тепловых процессов // Проблемы вычислительной и прикладной математики. — 2020. — № 3(27). — С. 76–89.

5. Normurodov C. B., Toyirov A. X., Yuldashev S. M. Numerical modeling of nonlinear wave systems by the spectral-grid method //International Scientific Journal Theoretical & Applied Science, Philadelphia, USA. – 2020. – Т. 83. – №. 3. – С. 43-54.
6. Narmuradov C. B. et al. MATHEMATICAL MODELING OF MOVEMENT OF A VISCOUS INCOMPRESSIBLE LIQUID BY THE SPECTRAL-GRID METHOD //Theoretical & Applied Science. – 2020. – №. 4. – С. 252-260.
7. Begaliyevich N. C., Khasanovich T. A. Spectral-grid method for solving evolution problems with high gradients //EPRA International Journal of Multidisciplinary Research (IJMR). – Т. 67.
8. Нармурадов Ч. Б., Тойиров А. Х. Математическое моделирование нелинейных волновых систем //Проблемы вычислительной и прикладной математики. – 2018. – №. 1. – С. 21-31.
9. BEGALIYEVICH N. C. et al. Mathematical Modeling of the Hydrodynamic Stability Problem by the Spectral-grid Method //International Journal of Innovations in Engineering Research and Technology. – Т. 7. – №. 11. – С. 20-26.
10. Toyirov A. K., Yuldashev S. M., Abdullayev B. P. Numerical modeling the equations of heat conductivity and burgers by the spectral-grid method //НАУКА 2020. ТЕОРИЯ И ПРАКТИКА. – 2020. – С. 30-31.
11. Нармурадов Ч. Б., Гуломкодиров К. А. Математическое моделирование уравнений Навье-Стокса в системе вихря и функции тока //Проблемы вычислительной и прикладной математики. – 2017. – №. 3. – С. 29-32.
12. Нармурадов Ч. Б., Холияров Э. Ч., Гуломкодиров К. А. Численное моделирование обратной задачи релаксационной фильтрации однородной жидкости в пористой среде //Проблемы вычислительной и прикладной математики. – 2017. – №. 2. – С. 12-19.
13. Abdirasulovna Z. S., Majidovna N. M. Evaluation of Errors in Numerical Solution of Problems //CENTRAL ASIAN JOURNAL OF MATHEMATICAL THEORY AND COMPUTER SCIENCES. – 2021. – Т. 2. – №. 9. – С. 45-47.
14. Abdirasulovna Z. S. Conducting a Computational Experiment using Test Functions //CENTRAL ASIAN JOURNAL OF MATHEMATICAL THEORY AND COMPUTER SCIENCES. – 2021. – Т. 2. – №. 9. – С. 51-53.
15. Shavkatovna D. Z. Solving Cauchy Problems Using Euler Methods Using the C# Programming Language and Method Mapping //International Journal of Innovative Analyses and Emerging Technology. – 2021. – Т. 1. – №. 4. – С. 74-77.
16. Abdirasulovna Z. S. Newton-Cotess Type Quadrature Formulas //CENTRAL ASIAN JOURNAL OF MATHEMATICAL THEORY AND COMPUTER SCIENCES. – 2021. – Т. 2. – №. 12. – С. 1-6.
17. Ochilov, A. (2012). Education and economic growth in Uzbekistan. *Perspectives of Innovations, Economics and Business, PIEB*, *12*(3), 21-33.
18. Ochilov, A. (2014). Is higher education a driving force of economic growth in Uzbekistan?. *Perspectives of Innovations, Economics and Business, PIEB*, *14*(4), 160-174.
19. Jurakulovna J. G. The Necessity and Theoretical Basis of Financial Statement Analysis in Modern Management //Academic Journal of Digital Economics and Stability. – 2021. – Т. 7. – С. 89-95.
20. Ochilov, A. O. (2017). The Higher Education Dynamics and Economic Growth: The Case of Uzbekistan. *Journal of Management Value & Ethics*, *7*(2), 46-53.
21. Ochilov, A. O. HIGHER EDUCATION IS AN IMPORTANT FACTOR IN STIMULATING ECONOMIC GROWTH. *GWALIOR MANAGEMENT ACADEMY*, *23*, 133.
22. Abitovna, K. N. (2020). Economic Mechanisms Of Formation And Use Of Intellectual Capital In The System Of Innovative Cooperation Of Education, Science And Production. *European Journal of Molecular & Clinical Medicine*, *7*(7), 929-937.