Multiplication Probability and Sum of Events, A Complete Group of Events, Absolute probability Formula

Gulkhayo Komolova
Teacher of Andijan Institute of Mechanical Engineering
komolovagulhayo@gmail.com

Olimova Barchinoy
Student of Andijan Institute of Mechanical Engineering

Annotation:
The article discusses probability theory difficulties such as determining the likelihood of a set of co-occurring, free, connected, and non-co-occurring occurrences, as well as the probability of non-co-occurring events.

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Co-occurring occurrences occur when the occurrence of one of two events in one experiment does not rule out the occurrence of the other.

Example. A — four points when the game is over; B — even number of points output. Events A and B together. Allow occurrences A and B to be supplied jointly, along with their probabilities and the likelihood that they will occur together.

Theorem. The probability that at least one of the two co-occurring events will occur is equal to the sum of these events' probabilities minus the probability that they will co-occur:

\[ P(A + B) = P(A) + P(B) - P(AB) \] (1)

Note 1. It's important to remember that events A and B might be both independent and connected when using the created formula.

For free events

\[ P(A + B) = P(A) + P(B) - P(A) \cdot P(B) \] ; (2)

For related events

\[ P(A + B) = P(A) + P(B) - P(A) \cdot P_a(B) \] (3)

Note 2. If events A and B do not coexist, then their occurrence together is a mathematical impossibility, and therefore \( P(AB) = 0 \). The formula for non-cohesive events is as follows:

\[ P(A + B) = P(A) + P(B) \]

Theorem. If A and B are random events that could happen as a result of the experiment,
\[ P(A + B) = P(A) + P(B) - P(AB) \quad (4) \]

**Proof:** It is clear that

\[ A + B = A + (B - AB) \]
\[ B = AB + (B - AB) \]

Non-reciprocal joins occur on the right side of these equations. The chance of a collection of non-co-occurring events is equal to the sum of the probabilities of the added events, according to Probability Axiom 3. As a result:

\[ P(A + B) = P(A) + P(B - AB) \]
\[ P(B) = P(AB) + P(B - AB) \]

From this

\[ P(A + B) - P(B) = P(A) - P(AB) \]

Or

\[ P(A + B) = P(A) + P(B) - P(AB) \]

is derived.

When events A and B are not mutually exclusive, formula (4) takes the form of

\[ P(A + B) - P(B) = P(A) - P(AB) \]

and is commonly referred to as the addition axiom.

For the case where the number of additions is n, formula (4) is generalized as follows:

\[
P\left( \sum_{k=1}^{n} A_k \right) = \sum_{k=1}^{n} P(A_k) - \sum_{1<i\leq n} P(A_iA_j) + \sum_{i<j<k\leq n} P(A_iA_jA_k) + ... + (-1)^{n-1} P(A_1A_2...A_n)
\]

For example, in the special case \( n = 3 \): The

\[
P(A_1 + A_2 + A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1A_2) - P(A_1A_3) - P(A_2A_3) + P(A_1A_2A_3)
\]

formula is appropriate

If the events \( A_1, A_2, ..., A_n \) are events that are not related together, then it is convenient to calculate the probability of the total event \( A_1 + A_2 + ... + A_n \) according to the following formula:

\[
P(A_1 + A_2 + ... + A_n) = 1 - P(\overline{A_1} \overline{A_2} ... \overline{A_n}) = 1 - P(\overline{A_1})P(\overline{A_2})...P(\overline{A_n})
\]

**Absolute Probability Formula.**

Suppose that \( A_1, A_2, ..., A_n \) are non-co-occurring events that may occur as a result of the observed experiment, let.
In this case, the following equation

$$A_1 + A_2 + \ldots + A_n = \Omega$$ \hfill (7)

In this case, the following equation

$$B = B\Omega = BA_1 + BA_2 + \ldots + BA_n$$ \hfill (8)

is valid for any event B that can occur in this experiment. The probability addition theorem is

$$P(B) = P(BA_1 + BA_2 + \ldots + BA_n) = \sum_{i=1}^{n} P(BA_i)$$

Applying the probability multiplication theorem for each of the $P(BA_i)$ terms, we obtain the equation

$$P(B) = \sum_{i=1}^{n} P(A_i)P(B/A_i)$$ \hfill (9)

This is called the full probability formula.

**Task 1.** 50% of the TV on sale are made in the first factory, 30% in the second factory and 20% in the third factory. 10% of the first factory TV, 8% of the second factory TV and 5% of the third factory TV have some kind of malfunction. Let's find out if a randomly selected TV set is perfect.

**Solution:** we can mark the following events:

- $B = \{TV \text{ on sale are perfect}\}$
- $A_i = \{The \ resulting \ TV \ is \ produced \ in \ the \ i-th \ factory\}$

Naturally, the events $A_1, A_2, A_3$ are not mutually exclusive and are $A_1 + A_2 + A_3 = \Omega$. Therefore, according to formula (9)

Depending on the circumstances

$$P(A_1) = 0.5; \quad P(A_2) = 0.3; \quad P(A_3) = 0.2$$

$$P(B/A_1) = 1 - 0.1 = 0.90 \quad P(B/A_2) = 1 - 0.08 = 0.92 \quad P(B/A_3) = 1 - 0.05 = 0.95$$

So,

$$P(B) = 0.5 \cdot 0.9 + 0.3 \cdot 0.92 + 0.2 \cdot 0.95 = 0.934.$$ 

**Answer:** $P(B)=0.934$.

**Task 2.** The shop has a number of machines. During a shift, the probability of needing to modify one machine is 0.2, and the probability of needing to adjust two machines is 0.13. During a shift, there’s a 0.07 chance of needing more than two machine modifications. Calculate the likelihood of the machines needing to be adjusted during the shift.

**Solution.** Let’s look at the following events:

- $A$ – the event that requires a single machine setup;
- $B$ – the event requires adjusting two machines;
- $C$ – A event that requires more than two machine settings;

$A, B, C$ – events are not mutually exclusive. We are interested in the following event: $(A + B + C)$— ABC machines that need to be adjusted during the shift:

$$P(A + B + C) = P(A) + P(B) + P(C) = 0.2 + 0.13 + 0.07 = 0.4$$

**Answer:** $P(A+B+C)=0.4$
**Task 3.** Unrelated to each other, the two hunters fired at the rabbit at the same time. The rabbit is shot if at least one of the hunters strikes the target. Find the probability of shooting the rabbit if the first hunter has a probability of reaching the target of 0.8 and the second has a probability of striking the target of 0.

**Solution.** Let’s look at the following events:

- **A** — the first hunter to hit the target;
- **B** — the second hunter hits the target.

**A and B are free events.** We are interested in: 

\((A + B)\) — incident.

\((A + B)\) — at least one hunter hits the target. In that case

\[ P(A + B) = P(A) + P(B) - P(A \cdot B) = 0.8 + 0.75 - 0.8 \cdot 0.75 = 0.95 \]

**Task 4.** The team consists of 12 athletes; five of them are masters in sports. Three athletes will be chosen at random from a hat. Check to see if all of the athletes you select are masters of their respective sports.

**Solution:**

- **A1** — first athlete — sports master;
- **A2** — second athlete — sports master;
- **A3** — third athlete — sports master;

\[ A = A_1 \cdot A_2 \cdot A_3 \] — uchalsportchi — sports master.

\( A_1, A_2, A_3 \) — related events. So,

\[ P(A) = P(A_1 \cdot A_2 \cdot A_3) = P(A_1) \cdot P(A_2) \cdot P(A_3) = \frac{5}{12} \cdot \frac{4}{11} \cdot \frac{3}{10} = \frac{1}{22} \]

**Answer:** \( P(A) = \frac{1}{22} \)

**References.**