Algebraic Topology Variations of Prime Order Automorphisms with Applications

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Annotation:
Simple polarized abelian kinds of odd integer dimensions have cyclic automorphism groups across Galois field. To create completely simple polarized algebraic variations of prime dimensions across finite fields, with a cyclic group of maximal order as their automorphism group. In this work, we focus on polarized Abelian types with an automorphism of prime order $s>2$. Convinced usual requirements on the algebraic expressions of its action on first-order differentials imply that such polarized varieties aren’t Jacobians of curvatures. There has been a lot of effort put into estimating the size of automorphism groups of broad categories. The latest articles, as well as the references they include, give a wealth of information on this subject.

1. Introduction

The thing we're interested in is looking at the prime number distributions, which can be represented as a property of the basis area of certain completely simple algebraic variety in our design. Let $G$ be a related simple Lie group with the Lie algebra $L$ as the appropriate Lie lattice. Let $L'$ be a maximum regular semi simple Lie subalgebra of $L$ having subgroup $G'$ as the corresponding subgroup. The centralizer of $G'$ in $G$ and its influence on the forms of the Lie algebra $L$ are the subjects of this work. These centralizers are abelian subgroup of $G$ in general. In the abelian group [1] provided the first precise account of continuously centralizers, wherever they exist, whereas [2] provided the first precise account of discrete centralizers [15-29].

Suppose $n \geq 1$ is an absolute value and $(\mathbb{Z}, \partial)$ a primarily polarized $n$—dimensional abelian variety over the complex number $\mathbb{C}$, an automorphism of $(\mathbb{Z}, \partial)$ that fulfills the cyclotomic expression $\sum_{j=0}^{s-1} \alpha^j = 0$ in $\text{End} \ Z$. In other terms, $\alpha$ is a regular automorphism with order $s$ with a limited set of established positions. This results in embeds $[30-45]$.

$$\mathbb{Z}(\omega_s) \hookrightarrow \text{End}(\mathbb{Z}), 1 \mapsto 1_{\mathbb{Z}}, \omega_s \mapsto \alpha,$$

$$\mathbb{Q}(\omega_s) \hookrightarrow \text{End}^0(\mathbb{Z}), 1 \mapsto 1_{\mathbb{Z}}, \omega_s \mapsto \alpha.$$
We describe some mathematical notations which is used above, such as \( \mathbb{C} \) is denoted by complex numbers, \( \mathbb{R} \) is denoted by field of a real number [46-77]. The field of rational numbers is represented by \( \mathbb{Q} \), and \( \mathbb{Z}_+ \) is represented by the set of nonnegative integers while \( \mathbb{Z} \) is the ring of integers. The following relationship of these numbers are well known in the literature [78-101].

\[
\mathbb{Z}_+ \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}
\]

For cardinality of \( B \), we write \( \#(B) \) if \( B \) is finite or maybe an empty set. Suppose \( s \) be an odd prime and \( \omega_s \in \mathbb{C} \) to be the primitive (complex) \( s \)th root of unity. For the \( s \)th cyclotomic ring and the \( s \)th cyclotomic field, we write \( \mathbb{Z}[\omega_s] \) and \( \mathbb{Q}(\omega_s) \) respectively. It forms the cyclic group \( \mu_s \) of \( p^s \) roots of unity of multiplicative order \( s \). We have \( \omega_s \in \mu_s \subset \mathbb{Z}[\omega_s] \subset \mathbb{Q}(\omega_s) \subset \mathbb{C} \).

Since the degree \([\mathbb{Q}(\omega_s):\mathbb{Q}] = s - 1\), it follows \((s - 1)|2n\). The \( n \)-dimensional complex vector space \( \mathbb{K}^1(Z) \) is operates on \( \mathbb{Q}(\omega_s) \) of variances of the first form on \( Z \) through functionality. The condition described in [9,10].

This gives \( \mathbb{K}^1(Z) \) the structure of \( \mathbb{Q}(\omega_s) \otimes_{\mathbb{Q}} \mathbb{C} \)-module. Obviously,

\[
\mathbb{Q}(\omega_s) \otimes_{\mathbb{Q}} \mathbb{C} = \bigoplus_{j=1}^{s-1} \mathbb{C}
\]

And the \( j \)th summand relates to the \( \mathbb{Q}(\omega_s) \mapsto \mathbb{C} \) field embed those transfers \( \omega_s \) to \( \omega_s^j \). As a result, \( \mathbb{Q}(\omega_s) \) operates on \( \mathbb{K}^1(Z) \) with multiplicities \( c_j(j = 1, \ldots, s - 1) \). All \( c_j \) are non-negative integers and \( \sum_{j=1}^{s-1} c_j = n \).

### 2. Principally polarized abelian varieties with automorphisms:

The fundamental findings are extended to mostly polarized abelian variants in this section. Direct approaches are almost certainly capable of improving real estimations [102-134]. A recent Feat finding, according to an argument of this rule, can be utilized to produce much developing work in characteristic 0. However, in the case of a good trait, such reasoning does not hold water [135-161].

In calculation,

\[
c_j + c_{s-j} = \frac{2n}{s-1} \text{ for all } j = 1, \ldots, s - 1; \tag{1}
\]

In general, this is a special case of a well-known outcome about endomorphism fields of complex abelian varieties [6].

On the finite cyclic group \( G = (\mathbb{Z}/s\mathbb{Z})^* \) of order \( (s - 1) \), We may view \( \{c\} \) as a nonnegative integer-valued function \( c = c_2 \), were

\[
c(j \mod s) = c_j(1 \leq j \leq s - 1), \sum_{h \in G} c(h) = n. \tag{2}
\]

The identity element \( 1 \mod s \) and the only element \((-1)mod s = (s - 1)mod s \) of order 2 are two distinguished elements contains in group \( G \). We write \(-h \) for the product \((-1)mod s \cdot h \) in \( G \), if \( h \) is an element of \( G \). If \( h = jmod s \) then \(-h = (s - j)mod s \). In light of (1),

\[
c_Z(h) + c_Z(-h) = \frac{2n}{s-1} \text{ for all } h \in G. \tag{3}
\]

**Definition 2.1.** Suppose a nonnegative integer-valued function to be \( l : G \rightarrow \mathbb{Z}_+ \). Then we say that \( l \) is admissible if

\[
c(h) + c(-h) = \frac{2n}{s-1} \quad \forall h \in G.
\]

The following ways is satisfied the above result

(i) In light of (3), our \( c = c_Z \) is admissible.
(ii) The number of admissible functions (forgiven n and p) is obviously
\[
\left( \frac{2n}{s-1} + 1 \right)^{(s-1)/2}
\]

**Example 2.2.** Assume thats = 3 and E be an elliptical curvature concluded C multiplied by \(\mathbb{Z}[\omega_3]\). We may take as E the smooth projective model of \(x^2 = z^3 - 1\) where \(\gamma_3\) acts on E by
\[\alpha_E: (z, x) \mapsto (z, \gamma_3x).\]
Clearly, \(\alpha_E\) satisfies the 3rd cyclotomic equation and respects the only principal polarization on E.

Let \(l(1)\) and \(l(-1)\)be nonnegative integers and n be a positive integer, whose sum is n. Let us put
\[X_1 = E^{l(1)}, X_2 = E^{l(2)}, X = X_1 \times X_2.\]
Let the principal polarization on Xis \(\partial_X\) that is the product of n pull-backs of the polarization on E. Let us consider the automorphism \(\alpha_3\) of Xthat act (diagonally) as \(\alpha_E\) on \(X_1 = E^{l(1)}\) and as \(\alpha_E^{-1}\) on \(X_2 = E^{l(2)}\).
Clearly, \(\alpha_3\) satisfies the 3rd cyclotomic equation and respects \(\partial_X\). It is also clear that
\[c_X(1) = l(1), c_X(1) = l(2).\]

We will also need the function
\[j: G = (\mathbb{Z}/s\mathbb{Z})^* \rightarrow \mathbb{Z}, (j \mod s) \mapsto j(1 \leq j \leq s - 1)\] \((4)\)

Clearly,
\[j(h_1h_2) \equiv j(h_1)j(h_2) \mod s \forall h_1, h_2 \in G\] \((5)\)
Recall that if \(l_1(h)\) and \(l_2(h)\)are composite-valued functions on G then for its convolution is the function \(l_1 \ast l_2(h)\)on G defined by
\[l_1 \ast l_2(h) = \frac{1}{s-1} \sum_{u \in G} l_1(u)l_2(u^{-1}h)\] \((6)\)

**Theorem 2.3.** Assume that \((Z, \partial)\) is the Jacobian of a smoothly complex type ncurvatureC with canonical primary polarisation. Then there will be a function with a positive integer value.

\[d: G = (\mathbb{Z}/s\mathbb{Z})^* \rightarrow \mathbb{Z} \subset \mathbb{C}\]
such that
\[\sum_{h \in G} d(h) = \frac{2n}{s-1} + 2,\] \((7)\)
\[c(v) = \frac{(s-1)}{s} \cdot d \ast f(-v) - 1 \forall v \in G.\] \((8)\)

**Proof.** Assume \((Z, \partial) \cong (f(C), \Theta)\) where \(f(C)\) is a Jacobian with standard primary polarization \(\Theta\) and C is an irreducible smooth flat dimension ncurvature. The Torelli theorem in Weil’s way \([11,12,14]\) implies the existence of an automorphism: \(\beta: C \rightarrow C\), which by functionality induces either \(\alpha\) or \(-\alpha\) on \(f(C) = Z\). We can and shall assume that induces by replacing \(\beta\) by \(\beta^{s+1}\) and keeping in mind that \((s + 1)\) is even and \(\alpha^s\) is an automorphism that is the identity of \(Z = f(C)\). \(\beta\) induces \(\alpha\) is the identity and \(\beta^a\) an automorphism of C since it produces the identical map on \(f(C)\) and \(n > 1\). The operation of \(\beta\) on C results in group insertion \([162-175]\).

\[\mu_s \mapsto Aut(C), \omega_s \mapsto \beta.\]
Assume $P \in C$ is a constant value of $\beta$. The $\beta$ automorphism of the appropriate one-dimensional curvature interplanetary $T_p^1(C)$ is then induced, which is multiplied by a complex number $\epsilon_p$. $\epsilon_p$ is a $s$th root of unity $[176-190]$.

**Corollary 2.4.** The smooth projective irreducible curvature is the quotient $D = C/\mu_s$. The degree of the map $C \to D$ is $s$, its implications vertices are precise duplicates of point sets of $\beta$, and all implications indicators are $s$.

**Lemma 2.5.** The projective line is biregularly isomorphic to $D$.

The proof of the above Lemma 2.5 Albanese functionally, the map $C \to D$ generates the surjective homomorphism of the respective jacobians $J(C) \to J(D)$, which slays the factors modules of the type $(Q) - (\beta(Q))$ for each $Q \in C(\mathbb{C})$. This means that it is lethal to $(1 - \alpha)(C)$, on either side, $1 - \alpha : J(C) \to J(C)$ is an isogeny. This suggests that the duplicate of $J(C)$ in $J(D)$ is zero, and thus $J(D) = 0$ due to surjectiveness. This suggests that $D$'s genus is zero.

**Corollary 2.6.** The fixed points of $\beta$ for the number $F(\beta)$ is $\frac{2n}{s-1} + 2$.

**Proof:** Applying the Riemann-Hurwitz’s formula for proof of the above Corollary to $C \to D$, we get

$$2n - 2 = s \cdot (-2) + (s - 1) \cdot F(\beta)$$

**Corollary 2.7.** Let $\beta^*: \mathbb{N}^1(C) \to \mathbb{N}^1(C)$ be the automorphism of the $g$-dimensional complex vector space $\mathbb{N}^1(C)$ induced by $\beta$ and $\tau$ the trace of $\beta^*$. Then

$$\tau = \sum_{j=1}^{s-1} c_j \omega_s^j = \sum_{h \in G} c(h) \omega_s^h.$$  

**Proof of the above Corollary consider the regular map $s$ and pick a $\beta$-invariant point $P_0$**

$$\psi : C \to J(C), Q \mapsto \alpha((Q) - (P_0))$$

The complex vector spaces is well-known that $\psi$ induces an isomorphism.

$$\psi^*: \mathbb{N}^1(J(C)) \cong \mathbb{N}^1(C)$$

Obviously,

$$\beta^* = \psi^* \alpha^* \psi^{*-1}$$

where $\alpha^*: \mathbb{N}^1(J(C)) = \mathbb{N}^1(J(C))$ denotes the $\mathbb{C}$-linear automorphism generated by $\alpha$. This means that the traces of $\beta^*$ and $\alpha^*$ do correspond. The definition of a $c_j$ now entails that the trace of $\beta^*$ equals $\sum_{j=1}^{s-1} c_j \omega_s^j$.

**Lemma 2.8.** Suppose a primitive $s$th root of unity is $\omega \in \mathbb{C}$. Then

$$\frac{1}{1-\omega} = -\frac{\sum_{j=1}^{s-1} j \omega^j}{s} = -\frac{\sum_{h \in G} j(h) \omega^h}{s}.$$  

**Proof.** We have

$$(1 - \omega) \left( \sum_{j=1}^{s-1} j \omega^j \right) = \sum_{j=1}^{s-1} (j \omega^j - j \omega^{j+1}) = \left( \sum_{j=1}^{s-1} \omega^j \right) - (s - 1) \omega^s = (-1) - (s - 1) = -s.$$
Ending the Theorem 1.4 proofs: Let $B$ denote the collection of specified points of $\beta$. We already know that 
$$\#(B) = \frac{2n}{s-1} + 2.$$ 
By using the holomorphic Lefschetz convergence point equations [1,2,5] to $\beta$,

$$1 - \bar{\tau} = \sum_{P \in B} \frac{1}{1 - \bar{\epsilon}_P}$$

(10)

where $\bar{\tau}$ is the complex conjugate of $\tau$. Recall that every $\epsilon_P$ is a (primitive) $p^{th}$ root of unity. Now Theorem 1.4 follows readily from the following assertion.

**Lemma 2.9.** Let us define for each $h \in G$ the nonnegative integer $d(h)$ as the number of fixed points $P \in B \subset C(\mathbb{C})$ such that $\epsilon_P = \omega^{sh}$. Then

$$\sum_{h \in G} d(h) = F(\beta) = \frac{2n}{s-1} + 2.$$ 

(11)

and

$$c(v) = \frac{(s-1)}{s} \cdot d * j(-v) - 1 \forall v \in G.$$ 

(12)

Proof: The equality (11) is obvious to prove Lemma 2.9. Let us prove (12). Combining (10) with Lemma 2.8 (applied to $\omega = \omega^h$ and Corollary 2.7) we get

$$1 - \sum_{h \in G} c(h) \omega^{-h} = \sum_{u \in G} d(u) \frac{1}{1 - \omega^u} = \frac{-1}{s} \left( \sum_{u \in G} d(u) \left( \sum_{h \in G} j(h) \omega^{hu} \right) \right)$$

$$= \frac{-1}{s} \sum_{v \in G} \left( \sum_{u \in G} d(u) j(u^{-1}v) \omega^v \right) = \frac{-1}{s} \sum_{v \in G} d * j(v) \omega^v$$

(here we use a substitution $v = hu$). Taking into account that

$$0 = 1 + \sum_{j=1}^{s-1} \omega^j = 1 + \sum_{v \in G} \omega^v,$$

we obtain

$$- \left( \sum_{v \in G} \omega^v \right) - \sum_{h \in G} c(h) \omega^{-h} = - \frac{(s-1)}{s} \sum_{v \in G} d * j(v) \omega^v.$$ 

Taking into account that the $(s-1)$-element set

$$\{\omega^j | 1 \leq j \leq s - 1\} = \{\omega^v | v \in G\}$$

we get $1 + c(-v) = (s-1)d * j(v)/s$, i.e.,

$$c(v) = \frac{(s-1)}{s} \cdot d * j(-v) - 1 \forall v \in G.$$ 

**Remark 2.10.** Let us consider the function

$$j_0 = j - \frac{s}{2}, G = (\mathbb{Z}/s\mathbb{Z})^\ast \rightarrow \mathbb{Q}, (j \mod s) \mapsto j - \frac{s}{2} \text{ where } j = 1, ..., s - 1$$

(13)

Then

$$j_0(-u) = -j_0(u) \forall u \in G.$$ 

(14)
We have
\[ d \ast j(v) = d \ast j_0(v) + \frac{s}{2(s-1)} \sum_{h \in G} d(h) = d \ast j_0(v) + \frac{s}{2(s-1)} \left( \frac{2n}{s-1} + 2 \right). \]
This implies that
\[ \frac{(s-1)}{s} \cdot d \ast j(v) = \frac{(s-1)}{s} \cdot d \ast j_0(v) + \frac{n}{s-1} + 1 \]
and therefore
\[ c(v) = \frac{(s-1)}{s} \cdot d \ast j_0(-v) + \frac{n}{s-1} \forall v \in G. \tag{15} \]
On the other hand, it follows from (14) that the convolution \( d \ast j_0 \) also satisfies
\[ d \ast j_0(-v) = d \ast j_0(v) \forall v \in G. \]
This implies that
\[ c(v) + c(-v) = \frac{(s-1)}{s} \cdot d \ast j_0(-v) + \frac{n}{s-1} + \frac{(s-1)}{s} \cdot d \ast j_0(v) + \frac{n}{s-1} = \frac{2n}{s-1} \forall v \in G. \tag{16} \]
Actually, we already know it see (1). It follows from (16) that
\[ c(v) = \frac{2n}{s-1} - \frac{(s-1)}{s} \cdot d \ast j(v) + 1 \forall v \in G \tag{17} \]
**Corollary 2.11.** We preserve Theorem 14’s terminology and assumptions. Let \( d' : G \rightarrow \mathbb{C} \) be a complex-valued function on \( G \) in the sense that
\[ c(v) = \frac{(s-1)}{s} \cdot d'(-v) - 1. \]
This implies that
\[ c(v) + c(-v) = \frac{(s-1)}{s} \cdot d'(-v) + \frac{n}{s-1} + \frac{(s-1)}{s} \cdot d'(v) + \frac{n}{s-1} = \frac{2n}{s-1} \forall v \in G. \]
Then the odd parts of functions \( d \) and \( d' \) do coincide, i.e.,
\[ d'(v) - d'(-v) = d(v) - d(-v) \]
In particular, if \( s = 3 \) then
\[ d'(v) = d(v) \forall v \in G. \]
Proof. If \( l : G \rightarrow \mathbb{C} \) is a complex-valued function on \( G \) and \( \chi : G \rightarrow \mathbb{C}^\ast \) is a character (homomorphism group) then we write
\[ a_\chi(l) = \frac{1}{s-1} \sum_{h \in G} l(h) \chi(h) \]
for the corresponding Fourier coefficient of \( l \). We have
\[ l(v) = \sum_{\chi \in G} a_\chi(l) \chi(v) \text{ where } \widehat{G} = \text{Hom}(G, \mathbb{C}^\ast) \tag{18} \]
Let us consider the function
\[ b : G \rightarrow \mathbb{C}, b(v) = d'(v) - d(v). \]
What we need to check is that
\[ b(v) = b(-v) \forall v \in G, \]
which means that for all odd characters \( \chi \) (i.e., characters \( \chi \) of \( G \) with
\[ \chi(-1 \text{mod} s) = -1 \]
the corresponding Fourier coefficient
\[ a_\chi(b) = 0. \]

It follows from (8) that \( b * j(-v) = 0 \) for all \( v \in G \), i.e.,
\[ b * j(v) = 0 \forall v \in G. \]

This implies that
\[ 0 = a_\chi(b * j) = a_\chi(b \cdot a_\chi(j)) \forall \chi \in (G) \wedge. \]

However, \( a_\chi(j) \neq 0 \) for all odd \( \chi \): it follows from [3,7,13]. This implies that \( a_\chi(b) = 0 \) for all odd \( \chi \). This ends the proof of the first assertion.

Now let \( s = 3 \). Then \( 2 + 2n/(s - 1) = n + 2 \) and \( G = \{1, -1\} \). We already know that \( d'(1) - d'(-1) = d(1) - d(-1). \)

Now has only to recall that \( d'(1) + d'(-1) = n + 2 = d(1) + d(-1). \)

**Remark 2.12.** If \( v \in G \) then there is an integer \( p_v \) that does not divide \( s \) and such that \( j(vh) - pj(h) \) is divisible by \( s \) for all \( h \in G \). Indeed, the function
\[ y : G \rightarrow (\mathbb{Z}/s\mathbb{Z})^*, h = j mod s \rightarrow j(h)mod s = jmod s \]
is a homomorphism group. Hence,
\[ y(vh) = gy(v)y(h) \forall v, h \in G. \]

Let us choose an integer \( p_v \in \mathbb{Z} \) such that
\[ p_v \mod \mathbb{Z} = y(h) = j(h) \mod \mathbb{Z}. \]

\( s \) does not divide \( p_v \) and
\[ j(vh)mod s = y(vh) = y(v) \cdot y(h) = (p_v mod s) \cdot y(h) = (p_v mod s) \cdot (j(h)mod s). \] This implies that \( j(vh) - pj(h) \) is divisible by \( s \) for all \( v \in G \).

**Corollary 2.13.** Let \( G \rightarrow \mathbb{Z} \) be an integer-valued function. Then conditions are equivalent as follows.

(i) \( a * h(1mod s) = \sum_{h \in G} a(h)j(h^{-1}) \in s\mathbb{Z}. \)

(ii) \( a * h(h) = \sum_{h \in G} a(h)j(v/h) \in s\mathbb{Z}v \in G. \)

**Proof.** Notice that in light of Remark 2.12 (applied to \( h^{-1} \)), if \( v \in G \) then there exists \( p_v \in \mathbb{Z} \) such that \( j(v/h) - pj(h) \) is divisible by \( s \) for all \( h \in G \). In other words, \( j(v/h) \equiv p_v(j(h)mod s \text{ and therefore} \)

\[ \sum_{h \in G} a(h)j(v/h) \equiv p_v \sum_{h \in G} a(h)j(h^{-1})mod s \forall v \in G. \]

This proof is completed.

3. A construction of Jacobians

Theorem 2.3 may be considered as an inverse of the following theorem.

**Theorem 3.1.** Consider \( n \) to be a positive integer, \( s \) odd prime, \( \omega_s \in \mathbb{C}a \text{ primitive } p^t \text{ root of unity, and } G = (\mathbb{Z}/s\mathbb{Z})^* \). Suppose that \( (s - 1)divides2n \). Let \( G \rightarrow \mathbb{Z}_4 \) be a non-negative integer-valued function such that

(i) \[ \sum_{h \in G} d(h) = \frac{2n}{s-1} + 2. \] (19)
\begin{equation}
\tag{20}
d \ast j(1 \text{mod } s) = \sum_{h \in G} d(h)j(h^{-1}) \in s\mathbb{Z}
\end{equation}
\begin{align*}
\text{Let } &\{l_h(z) \mid h \in G\}\text{bec}(s - 1)\text{-element set of mutually prime nonzero polynomials } l_h(z) \in \mathbb{C}[z] \text{that enjoy the following properties.} \\
&\text{(1) } \deg (l_h) = d(h) \text{ for all } h \in G. \text{ In particular, } l(z) \text{ is a (nonzero) constant polynomial if and only if } d(h) = 0. \\
&\text{(2) Each } l(z) \text{ has no repeated roots.}
\end{align*}

Let us consider a polynomial
\begin{align*}
l(z) &= l_d(z) = \prod_{h \in G} l_h(z)^{j(h^{-1})} \in \mathbb{C}[z]
\end{align*}
of degree $\sum_{h \in G} d(h)j(h^{-1})$. Suppose $C$ be the smooth projective model of the irreducible plane affine curvature
\begin{equation}
\tag{21}
x^2 = l_d(z)
\end{equation}
endowed with an automorphism $\alpha_C : C \to C$ induced by $(z, x) \mapsto (z, \omega_s x)$.

Suppose that the canonically principally polarized Jacobian of $C$ is $(J, \partial)$ endowed by the automorphism $\alpha$ induced by $\alpha_C$. Then $J$ and $\alpha$ enjoy the following properties.
\begin{align*}
&(a) \dim (J) = n \\
&(b) \sum_{j=0}^{s-1} \alpha^j = 0 \text{ in } \text{End} (J).
\end{align*}

Let $a : G \to \mathbb{Z}_s$ be the corresponding multiplicity function attached to the action of $\alpha$ on the differentials of the first kind on $\mathbb{Z}(2)$. Then
\begin{equation}
\tag{22}
c(\nu) = \frac{(s - 1)}{s} \cdot d \ast j(-\nu) - 1 \forall \nu \in G.
\end{equation}

Proof. If $\psi$ is a root of $l(z)$ then there is exactly one $h \in G$ that $\psi$ is a root of $l_h(z)$; in addition, the multiplicity of $\psi$ (viewed as a root of $l(z)$) is $j(h^{-1})$, which is not divisible by $s$. This infers that $l(z)$ is not a $p^{th}$ power in the polynomial ring $\mathbb{C}[z]$ and even in the field of rational function $\mathbb{C}(z)$. It follows from theorem 9.1 of [4] that the polynomial $x^2 - l(z) \in \mathbb{C}[z]$ is irreducible over $\mathbb{C}[z]$. This implies that the polynomial in two variables $x^2 - l(z) \in \mathbb{C}[z, x]$ is irreducible because every divisor that is a polynomial in $z$ is a constant. i.e., the affine plane curvature (21) is irreducible and its field of rational functions $K$ is the field of fractions of the domain

\begin{equation}
A = \mathbb{C}[z, x]/(x^2 - l(z))\mathbb{C}[z, x].
\end{equation}

Let smooth projective model of (21) be $C$. Then $K$ is the field $\mathbb{C}(C)$ of rational functions on $C$; in particular, $\mathbb{C}(C)$ is generated over $\mathbb{C}$ by rational functions $z, x$. Let $\pi : C \to \mathbb{P}^1$ be the regular map clear by rational function $z$. It has a degree $s$. Since
\begin{equation}
\deg(\pi) = \deg(l) = \sum_{h \in G} d(h)j(h^{-1})
\end{equation}
is divisible by \(s\), the map \(\pi\) is unramified at \(\infty\) (see Rejêt *8M* Sect. 4)) and therefore the set of branch points of \(\pi\) coincides with the set of roots of \(l(z)\).

The disjoint union of the sets \(R_h\) of roots of \(l_h(z)\). In particular, the number of branch points of \(\pi\) is

\[
\sum_{h \in G} \text{deg } (l_h) = \sum_{h \in G} d(h) = \frac{2n}{s-1} + 2.
\]

\(\pi\) is a Galois cover of degree \(s\), i.e., the field extension.

\[
\mathbb{C}(C)/\mathbb{C}(\mathbb{P}^1) = \mathbb{C}(C)/\mathbb{C}(\mathbb{P}^1)
\]

is a cyclic field extension of degree \(s\). In addition, the cyclic Galois group \(Gc1(\mathbb{C}(C)/\mathbb{C}(\mathbb{P}^1))\) is generated by the automorphism \(\alpha_C : C \to C\) induced by \(\alpha_C : C \to C, (z, x) \mapsto (z, \omega_s x)\).

It follows from the Riemann-Hurwitz formula [8] that the genus of \(C\) is

\[
\frac{\left(\frac{2n}{s-1} + 2\right) - 2}{2}(s - 1) = n.
\]

In addition, the automorphism \(\alpha\) of the polarized jacobian \((J, \partial)\) induced by \(\alpha_C\) satisfies the \(s\)th cyclotomic equation

\[
\sum_{j=0}^{s-1} \alpha^j = 0 \text{ in End } (J)
\]

Suppose that the set of ramification points of \(\pi\) is \(B \subset C(\mathbb{C})\). \(B\) accords with the usual of static points of \(\alpha_C\). The map \(z : C(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C})\) establishes a bijection between \(B\) and the disjoint union of all \(R_h\). Let us putting

\[
B_h = \{P \in B|z(P) \in R_h\}.
\]

Then \(B\) partitions onto a disjoint union of all \(B_h\)'s and

\[
\#(B_h) = \text{deg } (l_h) = d(h) \forall h \in G.
\]

Let \(P \in B\). The action of \(\alpha\) on the tangent space to \(C\) at \(P\) is multiplication by a certain \(p^{th}\) root of unity \(\varepsilon_p\). We observe and claim that \(\varepsilon_p = \omega_s^{j(h)}\) if \(P \in R_h\).

Indeed, we have

\[
z(P) = \psi \in R_h, x(P) = 0.
\]

Let

\[
ord_p : \mathbb{C}(C) \to \mathbb{Z}
\]

be the discrete valuation map attached to \(P\). Then one may easily check that

\[
ord_p(z - \psi) = s, ord_p(z - \xi) = 0 \forall \xi \in \mathbb{C}\setminus\psi.
\]

This implies that

\[
s \cdot j(h^{-1}) \cdot ord_p(z - \psi) = ord_p(x^s) = s \cdot ord_p(x)
\]
and therefore
\[ \text{ord}_p(x) = j(h^{-1}) \quad (22) \]
In light of (5), there is an integer \( m \) such that
\[ j(h^{-1}) \cdot j(h) = 1 + sm. \]
Combining this with (22), we obtain that
\[ \text{ord}_p \left( \frac{x^{j(h)}}{(z - \psi)^m} \right) = j(h^{-1}) \cdot j(h) - sm = 1 \]
and therefore \( t = x^{j(h)}/(z - \psi)^m \) is a local parameter of \( C \) at \( P \). The action of \( \alpha \) multiplies \( t \) by \( \omega_z^{j(h)} \) and therefore \( \varepsilon_p = \omega_z^{j(h)} \), which proves the Claim.

Now the desired result follows from Proposition 1.11 applied to \( Z = J, \beta = \alpha \).

**Example 3.2.** Let \( s = 3 \). The number of admissible functions is \((n + 1)\).

Let us list all the possibilities for \( a \) when \( n \) are given. Let us identify
\[ G = (\mathbb{Z}/3\mathbb{Z})^* = \{1 \text{mod } 3, 2 \text{mod } 3\} \]
with the set \( \{1, 2\} \) in an obvious way. We have the following conditions on \( d \).
\[ d(1), d(2) \in \mathbb{Z}_+, d(1) + d(2) = n + 2, 3|d(1) + 2d(2)) \]
The congruence condition means that \( d(1) \equiv d(2) \text{mod } 3 \). So, the conditions on \( d \) are as follows.
\[ d(1), d(2) \in \mathbb{Z}_+, d(1) + d(2) = n + 2, d(1) \equiv d(2) \text{mod } 3 \]
The list (and number) of corresponding \( a \) depends on \( n \text{mod } 3 \). Namely, there are the natural three cases.

(i) \( n \equiv 1 \text{mod } 3 \), i.e., \( n = 3p + 1 \) where \( p \) is a nonnegative integer. Then
\[ d(1) + d(2) = n + 2 = 3p + 3 = 3(p + 1), \]
and therefore both \( d(1) \) and \( d(2) \) are divisible by 3. Hence there are exactly \((p + 2)\) options for \( d \), namely,
\[ d(1) = 3b, d(2) = 3(p + 1 - b); b = 0, \ldots, (p + 1) \quad (23) \]
The corresponding \( a \) are as follows (where \( b = 0, \ldots, (p + 1) \)
\[ c(2) = \frac{1}{3}(d(1) + 2d(2)) - 1 = b + 2 \cdot (p + 1 - b) - 1 \]
\[ = b + 2(p + 1 - b) - 1 = (2p + 1) - b; \]
\[ c(1) = \frac{1}{3}(2d(1) + d(2)) - 1 = 2b + (p + 1 - b) - 1 = p + b. \]
So, we get
\[ c(1) = p + b, c(2) = (2p + 1) - b; b = 0, \ldots, p + 1. \]
The number of \( a \)’s is
\[ p + 2 = \frac{n + 5}{3}. \]

(ii) \( n \equiv 2 \mod 3 \), i.e., \( n = 3p + 2 \). Then
\[
d(1) + d(2) = n + 2 = 3p + 4 = 3(p + 1) + 1,
\]
In above \( p \) is a non-negative integer and therefore both \( d(1) - 2 \) and \( d(2) - 2 \) are divisible by 3. Hence there are exactly \((p + 1)\) options for \( d \), namely,
\[
d(1) = 3b + 2, d(2) = 3(p - b) + 2; (b = 0, ..., p)
\]
The corresponding \( a \) are as follows (where \( b = 0, ..., p \)).
\[
c(2) = \frac{1}{3} (d(1) + 2d(2)) - 1 = b + 2(p - b) + 2 - 1 = (2p + 1) - b;
\]
\[
c(1) = \frac{1}{3} (2d(1) + d(2)) - 1 = 2b + (p - b) + 2 - 1 = (p + 1) + b = p + b.
\]
So, we get
\[
c(1) = (p + 1) + b, c(2) = (2p + 1) - b; b = 0, ..., p.
\]
The number of \( a \)'s is
\[
p + 1 = \frac{n + 1}{3}.
\]
(iii) \( n \equiv 0 \mod 3 \), i.e., \( n = 3p \). Then
\[
d(1) + d(2) = n + 2 = 3p + 2,
\]
In above \( p \) is a non-negative integer and therefore both \( d(1) - 1 \) and \( d(2) - 1 \) are divisible by 3. Hence there are exactly \((p + 1)\) options for \( d \), namely,
\[
d(1) = 3b + 1, d(2) = 3(p - b) + 1; (b = 0, ..., p)
\]
The corresponding \( a \) are as follows (where \( b = 0, ..., p \)).
\[
c(2) = \frac{1}{3} (d(1) + 2d(2)) - 1 = b + 2(p - b) + 1 - 1 = 2p - b;
\]
\[
c(1) = \frac{1}{3} (2d(1) + d(2)) - 1 = 2b + (p - b) + 1 - 1 = p + b.
\]
So, we get
\[
c(1) = p + b, c(2) = 2p - b; b = 0, ..., p. \tag{24}
\]
The number of \( a \)'s is
\[
p + 1 = \frac{n + 3}{3}. \tag{25}
\]
The above result shows that where \( p \) is a non-negative integer congruence condition are satisfies.

4. Conclusion
The Dynein network of $\mathcal{L}$ can be used to represent the most periodic reduced subalgebras of a simple Lie algebra $\mathcal{L}$. Clearly, any such subalgebra arises as a semi simple Lie algebra with a Dynein diagram obtained by deleting one node from the Dynein circuit of $\mathcal{L}$ with a mark equal to 1 direct sum and asymmetric subalgebra composed of the intersect of the husks of the remaining roots. Finally, the well-known constraints on the algebraic formulations of its action on first-order distinctions suggest that such polarized varieties are not Jacobians of curvatures.

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