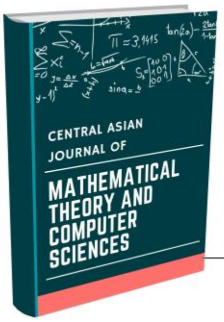

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Using the Maple System in Selecting an Efficient Model for the Analysis of Experimental Results

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Annotation:

At present, any serious statistical calculations, as a rule, are carried out on computers and, above all, on personal computers. This paper shows the quality and efficiency of mathematical models based on the experimental results of solving engineering and economic problems using the Maple program, as well as their significance in the analysis and decision-making using digital methods [1,2,3,7,8,9].

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INTRODUCTION. The mathematical model of the experimental results of regression analysis in the processing of statistical data was used to construct the regression equation using the least squares method and to assess the quality, adequacy and value of regression coefficients of these models [5,8,9]. In solving these problems, the model is based on the formula for the approximate approximation of the error in the assessment of quality, the adequacy of the model-regression equation from Fisher's F-criterion, the value of regression coefficients from the Student's criterion. The methods and programs in the Maple system were used to solve all these problems [1,2,3,5,6,10,11].

MAIN PART. The results of experiments to determine the relationship between the number of products sold as a result of observations and the amount of x spent on its advertising are as follows.

x	1.5	4.0	5.0	7.0	8.5	10.0	11.0	12.5
y	5.0	4.5	7.0	6.5	9.5	9.0	11.0	9.0

Based on the results of this experiment, it is important to determine the best-effective relationship-regression equation for the analysis and draw conclusions and decisions based on it. To do this, we solve the problem of choosing the most effective of the linear, logarithmic and hyperbolic connections based on the results of experiments found in the method of small squares [4]:

1.We use the following system to determine the parameters a, b of the linear regression equation $\tilde{y} = ax + b$:

$$\begin{cases} \left(\sum_{i=1}^{n} x_i^{\,2}\right)a + \left(\sum_{i=1}^{n} x_i\right)b = \sum_{i=1}^{n} x_i y_i \\ \left(\sum_{i=1}^{n} x_i\right)a + nb = \sum_{i=1}^{n} y_i \end{cases} \qquad (1)$$

2. We use the following system to determine the a, b parameters of the logarithmic regression equation $\tilde{y} = a\ln(x) + b$:

$$\begin{cases} a\,n + b\sum_{i=1}^{n}\ln(x_i) = \sum_{i=1}^{n} y_i \\ a\sum_{i=1}^{n}\ln(x_i)^2 + b\sum_{i=1}^{n}\ln(x_i) = \sum_{i=1}^{n} y_i \ln(x_i) \end{cases} \qquad (2)$$

3. We use the following system to determine the a, b parameters of the $\tilde{y} = a + \frac{b}{x}$ hyperbolic regression equation:

$$\begin{cases} a\,n + b\sum_{i=1}^{n}\frac{1}{x_i} = \sum_{i=1}^{n} y_i \\ a\sum_{i=1}^{n}\frac{1}{x_i} + b\sum_{i=1}^{n}\frac{1}{x_i^2} = \sum_{i=1}^{n}\frac{y_i}{x_i} \end{cases} \qquad (3)$$

To determine the $\tilde{y} = ax + b$, $\tilde{y} = a\ln(x) + b$, $\tilde{y} = a + \frac{b}{x}$ connections found in the above systems (1), (2), (3), and for each connection, we perform all of the following statistical estimates based on the Maple program.

1. **Evaluation of model quality.** To evaluate the quality of the model, the average approximation error is determined. We determine the average deviation (approximation) of the calculated data from the actual data by the following formula:

$$\bar{A} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{y_i - \tilde{y}_i}{y_i}\right| \cdot 100\% .$$

The smaller these deviations, the closer the found values are to the theoretical values. This indicates the quality-efficiency of the model. An average error of 8-10%, or no more than 12-15% rest, determines the quality of the model.

2. **Evaluation of model adequacy.** We determine the adequacy of the regression equation based on Fisher's F-criterion with the following formula:

$$F = \frac{\bar{S}_y^2}{\bar{S}_{qold}^2}$$

here $\bar{S}_y^2 = \frac{1}{k_1}\left(\sum_{i=1}^{n} y_i - \frac{1}{n}\left(\sum_{i=1}^{n} y_i\right)^2\right)$ - total variance, $\bar{S}_{qold}^2 = \frac{1}{k_2}\sum_{i=1}^{n}\left(y_i - \tilde{y}\right)^2$ - residual dispersion. The regression equation determines how many times the experimental results $\bar{y}$ are better than the average. We find it from the ratio of the above variances. The value of this criterion F is compared with the value $F_{tab}(\alpha,k_1,k_2)$ in the application table, where α- value, $k_1=n-1$, $k_2=n-m-1$ degrees of freedom, n-number of experiments, m- the number of parameters belonging to x. In this case:

-if the calculation result is $F>Ftab$, the regression equation found is adequate (statistically significant);

-if $F<Ftab$, the regression equation is not adequate.

3. Evaluation of the value of regression coefficients.

According to the student's criterion, we evaluate the values of *a* and *b* of the coefficients by comparing their random errors with m_a and m_b:

$$t_a = \frac{a}{m_a},\ t_b = \frac{b}{m_b}$$

here $m_a = \frac{S_{qold}}{\sigma_x\sqrt{n}}$, $m_b = \frac{1}{n\sigma_x} S_{qold}\sqrt{\sum_{i=1}^{n} {x_i}^2}$

here $\bar{S}_{qold}^2 = \frac{1}{n-2}\sum_{i=1}^{n}\left(y_i - \tilde{y}\right)^2$ - residual dispersion and $\sigma_x = \sqrt{\frac{1}{n}\sum_{i=1}^{n} {x_i}^2 - \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2}$ -o‘rtacha kvadratik og‘ishma. Bu t_a, t_b lar $v=n-2$ erkinlik darajasi bo‘yicha Styudent taqsimotiga bo‘ysunadi. Styudent kriteriyasi uchun tuzilgan taqsimot jadvalidan kritik qiymatni $t_{kr}=t(\alpha,v)$ kabi topamiz:

standard deviation. These units, t_a, t_b are subject to the Student's distribution according to the degree of freedom $v=n-2$ From the distribution table constructed for the student criterion, we find the critical value as $t_{kr}=t(\alpha,v)$:

1) if $t_a>t_{kr}$, parameter *a* is statistically valuable, otherwise it is not valuable;

2) If $t_b>t_{kr}$, parameter *b* is statistically significant, otherwise it is not.

Now to find the solution of the problem of determining the quality, adequacy, value of the coefficients of the linear regression equation (model) found on the basis of the table compiled on the above $n=8$ observations, directly below Maple program.

1. 1. A straight line is connected according to the above rule:

Maple software

> restart; with(stats): Digits:=5: n:=8:

Values found on the basis of experimental results:

> x1:=1.5:x2:=4:x3:=5:x4:=7:x5:=8.5:x6:=10:x7:=11:x8:=12.5:

> y1:=5:y2:=4.5:y3:=7:y4:=6.5:y5:=9.5:y6:=9:y7:=11:y8:=9:

Random quantities:

> X:=Vector([x1,x2,x3,x4,x5,x6,x7,x8],datatype=float):

> Y:=Vector([y1,y2,y3,y4,y5,y6,y7,y8],datatype=float):

Determining the coefficients and equations of linear correlation of random variables:

> SX:=add(X[i],i=1..n); $SX := 59.500$

> SY:=add(Y[i],i=1..n); $SY := 61.500$

> SXX:=add(X[i]^2,i=1..n); $SXX := 541.75$

> SYY:=add(Y[i]^2,i=1..n); $SYY := 509.75$

> SXY:=add(X[i]*Y[i],i=1..n); $SXY := 510.25$

Determining the coefficients of the straight line correlation equation (1):

> ab:=solve({a*SX+n*b=SY,a*SXX+b*SX=SXY},{a,b});

$ab := \{a = 0.53260, b = 3.7263\}$

Determine the equation of a straight line connection:

> yt:=ab[1]*x+ab[2]; $yt := x\,a + b = 0.53260\,x + 3.7263$

> a:=.532; b:=3.726; $a := 0.532 \quad b := 3.726$

The value and sum of the straight line connections:

> Yt:={seq(a*X[i]+b,i=1..n)};

$Yt := \{4.5240, 5.854, 6.386, 7.450, 8.2480, 9.046, 9.578, 10.376\}$

> SXY2:=add((Y[i]-a*X[i]-b)^2,i=1..n); $SXY2 := 8.8245$

The average deviation and sum of the experimental values from the straight line connection:

> A:=seq(abs((Y[i]-a*X[i]-b)/Y[i]),i=1..n): A:=evalf(%,4)*100;

$A := 9.49900, 30.1400, 8.73100, 14.6700, 13.1300, 0.570000, 12.8700, 15.3600$

> SA:=add(abs((Y[i]-a*X[i]-b)/Y[i]),i=1..n)*100; $SA := 104.90$

Let's create a table to determine the values corresponding to the following variables and their sum:

```
> u:=array(1..10,1..8):
> for i to n do for j to n do
u[1,1]:="X[i]";u[1,2]:="Y[i]";u[1,3]:="X[i]^2";
u[1,4]:="Y[i]^2"; u[1,5]:="X[i]*Y[i]"; u[1,6]:="a*X[i]+b"; u[1,7]:="(Y[i]-a*X[i]-b)^2";
 u[1,8]:="A[i]"; u[i+1,1]:=X[i];u[i+1,2]:=Y[i];u[i+1,3]:=X[i]^2; u[i+1,4]:=Y[i]^2; u[i+1,5]:=X[i]*Y[i];
 u[i+1,6]:=a*X[i]+b; u[i+1,7]:=(Y[i]-a*X[i]-b)^2; u[i+1,8]:=A[i];
u[n+2,1]:=SX; u[n+2,2]:=SY; u[n+2,3]:=SXX; u[n+2,4]:=SYY;u[n+2,5]:=SXY; u[n+1,6]:=a*X[i]+b;
u[n+2,6]:=`-`; u[n+2,7]:=SXY2; u[n+2,8]:=SA;
od;od; evalf(evalm(u),5);
```

"X[i]"	"Y[i]"	"X[i]^2"	"Y[i]^2"	"X[i]*Y[i]"	"a*X[i]+b"	"(Y[i]-a*X[i]-b)^2"	"A[i]"
1.5000	5.	2.2500	25.	7.5000	4.5250	0.22558	9.4990
4.	4.5000	16.	20.250	18.	5.8563	1.8396	30.140
5.	7.	25.	49.	35.	6.3888	0.37356	8.7310
7.	6.5000	49.	42.250	45.500	7.4538	0.90973	14.670
8.5000	9.5000	72.250	90.250	80.750	8.2526	1.5561	13.130
10.	9.	100.	81.	90.	9.0513	0.0026317	0.57000
11.	11.	121.	121.	121.	9.5838	2.0056	12.870
12.500	9.	156.25	81.	112.50	10.383	1.9114	15.360
59.500	61.500	541.75	509.75	510.25	`-`	8.8242	104.98

From the values of these variables and the sum of the last line, we use the linear relationship in statistical estimates:.

1) *The effectiveness of the connection found:*

> At:=add(abs((Y[k]-a*X[k]-b)/Y[k]),k=1..n)/n; $At := 0.13112$

> At:=At*100; $At := 13.112$

Average deviation of values found in the experiment with regression correlation: 13.12%. If the approximation error is not more than 12-15%, the structured link model is a good module and allows to estimate (predict) the calculations in advance.

2) *Assess the adequacy of the bond: the number of n-experiments and the degrees of freedom k_1, k_2:*

> n:=8.: m:=1: k1:=n-1; k2:=n-m-1; $k1 := 7.$ $k2 := 6.$

General dispersion:

> S2y:=(SYY-SY^2/n)/k1; $S2y := 5.2814$

Residual dispersion:

> S2qol:=add((Y[k]-a*X[k]-b)^2,k=1..n)/k2; $S2qol := 1.4708$

F-Fisher criterion:

> F:=S2y/S2qol; $F := 3.5908$

According to the Fisher table *Ftab*(α,k1,k2)=*Ftab*(0.05,7,6)=4.21

> Ftab:=4.21: #F<Ftab linear connection is adequate

3) *Evaluation of the value of regression coefficients:*

Find the random error of the coefficients:

> sigma[x]:=sqrt(SXX/n-(SX/n)^2); $\sigma_x := 3.5218$

> ma:=S2qol/(sigma[x]*sqrt(n)); $ma := 0.14766$

> mb:=S2qol*sqrt(SXX)/(n*sigma[x]); $mb := 1.2151$

We evaluate the value of the found regression coefficients with their random error according to the Student's criterion:

> a:=.5325984252; b:=3.726299213; $a := 0.5325984252$ $b := 3.726299213$

> ta:=a/ma; $ta := 3.6069$

> tb:=b/mb; $tb := 3.0667$

From the distribution table (appendix) on the student's criterion we get the critical value: tkr=$t(\alpha,\nu)=t(\alpha,n-2)=t(0.05, 6)=2.45$

In this case, we see that the coefficients a and b are statistically significant, since $t_a>t_{kr}$, $t_b>t_{kr}$:

> tkr:=2.45:

> ta-tkr>0; $0 < 1.1569$

> tb-tkr>0; $0 < 0.6167$

This means that the linear relationship formed for the experimental results is completely adequate, and we found that all its coefficients are valuable.

2. The logarithmic relationship $y=a\cdot ln(x)+b$ **is based on the above rule:**

Maple software

> restart; with(stats):

> Digits:=6: n:=8:

> x1:=1.5:x2:=4:x3:=5:x4:=7:x5:=8.5:x6:=10:x7:=11: x8:=12.5:

> y1:=5:y2:=4.5: y3:=7:y4:=6.5: y5:=9.5: y6:=9: y7:=11: y8:=9:

> X:=Vector([x1,x2,x3,x4,x5,x6,x7,x8],datatype=float):

> Y:=Vector([y1,y2,y3,y4,y5,y6,y7,y8],datatype=float):

> {seq(X[i],i=1..n)}: evalf(%,4); SX:=add(X[i],i=1..n);

$\{1.500, 4., 5., 7., 8.500, 10., 11., 12.50\}$ $SX := 59.5000$

> SXX:=add(X[i]^2,i=1..n); $SXX := 541.750$

> {seq(Y[i],i=1..n)}: evalf(%,4); SY:=add(Y[i],i=1..n);

$\{4.500, 5., 6.500, 7., 9., 9.500, 11.\}$ $SY := 61.5000$

> SYY:=add(Y[i]^2,i=1..n); $SYY := 509.750$

> seq(ln(X[i]),i=1..n): evalf(%,4);

$0.4055, 1.386, 1.609, 1.946, 2.140, 2.303, 2.398, 2.526$

> SlnX:=add(ln(X[i]),i=1..n); $SlnX := 14.7134$

> seq(ln(X[i])^2,i=1..n): evalf(%,4);

$0.1644, 1.922, 2.590, 3.787, 4.580, 5.302, 5.750, 6.379$

> SlnXX:=add((ln(X[i])^2),i=1..n); $SlnXX := 30.4741$

> {seq(Y[i]*ln(X[i]),i=1..n)}: evalf(%,4);

$\{2.027, 6.238, 11.27, 12.65, 20.33, 20.72, 22.73, 26.38\}$

> SYlnX:=add(Y[i]*ln(X[i]),i=1..n); $SYlnX := 122.342$

Determining the coefficients of the logarithmic correlation equation (2):

> ab:=solve({a*SlnX+n*b=SY,a*SlnXX+b*SlnX=SYlnX},{a,b}); $ab := \{a = 2.70471, b = 2.71307\}$

> yt:=ab[1]*ln(x)+ab[2]; $yt := \ln(x)\ a + b = 2.70471 \ln(x) + 2.71307$

We set *a* and *b* in the system solution to *y=aln(x)+b ga qo'yamiz*:

> a:=2.70471; b:=2.71307; $a := 2.70471 \quad b := 2.71307$

> Yt:={seq(a*ln(X[i])+b,i=1..n)}: evalf(%,4);

{3.810, 6.463, 7.066, 7.976, 8.501, 8.941, 9.199, 9.544}

> seq((Y[i]-a*ln(X[i])-b)^2,i=1..n)

1.41672, 3.85176, 0.00437355, 2.17914, 0.997346, 0.00349341, 3.24475, 0.296408

> SYYt2:=add((Y[i]-a*ln(X[i])-b)^2,i=1..n); $SYYt2 := 11.9940$

> A:=seq(abs((Y[i]-a*ln(X[i])-b)/Y[i]),i=1..8): A:=evalf(%,4);

$A := 0.2381, 0.4361, 0.009448, 0.2271, 0.1051, 0.006567, 0.1638, 0.06049$

> SA:=add(abs((Y[i]-a*ln(X[i])-b)/Y[i]),i=1..n)*100;

$SA := 124.668$

> #At:=add(abs((Y[i]-a*X[i]-b)/Y[i]),i=1..n)*100/n;

> u:=array(1..10,1..8):

> for i to n do for j to n do

u[1,1]:="X[i]";u[1,2]:="Y[i]";u[1,3]:="ln(X[i])";

u[1,4]:="ln(X[i])^2";

u[1,5]:="ln(X[i])*Y[i]"; u[1,6]:="a*ln(X[i])+b"; u[1,7]:="(Y[i]-a*ln(X[i])-b)^2"; u[1,8]:="A[i]";

u[i+1,1]:=X[i];u[i+1,2]:=Y[i];u[i+1,3]:=ln(X[i]); u[i+1,4]:=ln(X[i])^2;

u[i+1,5]:=ln(X[i])*Y[i]; u[i+1,6]:=a*ln(X[i])+b; u[i+1,7]:=(Y[i]-a*ln(X[i])-b)^2; u[i+1,8]:=A[i];

u[n+2,1]:=SX;u[n+2,2]:=SY; u[n,6]:=a*ln(X[i])+b;u[n+2,6]:=\`-\`; u[n+2,3]:=SlnX; u[n+2,4]:=SlnXX;

u[n+2,5]:=SYlnX; u[n+2,7]:=SYYt2; u[n+2,8]:=SA;

od;od; evalf(evalm(u),5);

"X[i]"	"Y[i]"	"ln(X[i])"	"ln(X[i])^2"	"ln(X[i])*Y[i]"	"a*ln(X[i])+b"	"(Y[i]-a*ln(X[i])-b)^2"	"A[i]"
1.5000	5.	0.40547	0.16440	2.0273	3.8097	1.4167	0.2381
4.	4.5000	1.3863	1.9218	6.2383	6.4626	3.8518	0.4361
5.	7.	1.6094	2.5903	11.266	7.0661	0.0043736	0.009448
7.	6.5000	1.9459	3.7866	12.648	7.9762	2.1791	0.2271
8.5000	9.5000	2.1401	4.5799	20.331	8.5013	0.99735	0.1051
10.	9.	2.3026	5.3019	20.723	8.9409	0.0034934	0.006567
11.	11.	2.3979	5.7499	26.377	9.5444	3.2448	0.1638
12.500	9.	2.5257	6.3793	22.732	9.5444	0.29641	0.06049
59.500	61.500	14.713	30.474	122.34	\`-\`	11.994	124.67

We use the logarithmic relationship of the values of these variables and the sum of the last line in statistical estimates:.

1) The effectiveness of the found link:

> add((Y[k]-a*ln(X[k])-b)^2,k=1..n); 11.9940

> At:=add(abs((Y[k]-a*ln(X[k])-b)/Y[k]),k=1..n)/n;

$At := 0.155835$

> At:=At*100; $At := 15.5835$

Average deviation of values found in the experiment with regression correlation: 13.12%. A structured module with an approximation error of no more than 12-15% is a good module for predicting computational calculations.

2) Assess the adequacy of the connection:

n is the number of tests.

> n:=8.: m:=1: k1:=n-1; k2:=n-m-1; $k1 := 7.$ $k2 := 6.$

General dispersion:

> S2y:=(SYY-SY^2/n)/k1; $S2y := 5.28129$

Residual dispersion:

> S2qol:=add((Y[k]-a*ln(X[k])-b)^2,k=1..n)/k2;

$S2qol := 1.99900$

F-Fisher criterion:

> F:=S2y/S2qol; $F := 2.64197$

According to Fisher's schedule Ftab(α,*k1*,*k2*)=*Ftab*(0.05,7,6)=4.21

> Ftab:=4.21: #F<Ftab the logarithmic relationship is adequate

> F<Ftab:#(2.64<4.21)

3) Evaluation of the value of regression coefficients:

Find the random error of the coefficients:

> sigma[x]:=sqrt(SXX/n-(SX/n)^2); $\sigma_x := 3.52170$

> ma:=S2qol/(sigma[x]*sqrt(n)); $ma := 0.200685$

> mb:=S2qol*sqrt(SXX)/(n*sigma[x]) $mb := 1.65146$

We evaluate the value of the found regression coefficients with their random error according to the Student's criterion:

> #a:=.5325984252; b:=3.726299213;

> ta:=a/ma; $ta := 13.4774$

> tb:=b/mb; $tb := 1.64283$

From the Student Criteria Distribution Table (Appendix) we get the critical value: tkr=*t*(α,ν)=*t*(α,n-2)=*t*(0.05, 6)=2.45.

From this:

> tkr:=2.45:

> ta-tkr>0; $0 < 11.0274$ **# a statistically valuable**

> tb-tkr>0; $0 < -0.80717$ **# b not statistically valuable**

From this we see that the coefficients a are statistically significant and b are not statistically significant. So, the linear relationship model is completely adequate to Fisher's F-criterion, and we found that some of its coefficients are not values. The model found can be used to draw conclusions and make decisions based on the results of a given experiment, but it cannot be used to make predictions.

3. The hyperbolic relationship *y=a/x+b* is based on the above rule:

Maple software

> restart; with(stats): Digits:=6: n:=8:

> x1:=1.5:x2:=4: x3:=5:x4:=7: x5:=8.5:x6:=10:x7:=11:x8:=12.5:

> y1:=5:y2:=4.5: y3:=7:y4:=6.5: y5:=9.5: y6:=9: y7:=11:y8:=9:

> X:=Vector([x1,x2,x3,x4,x5,x6,x7,x8],datatype=float):

> Y:=Vector([y1,y2,y3,y4,y5,y6,y7,y8],datatype=float):

> {seq(X[i],i=1..n)}: evalf(%,4); SX:=add(X[i],i=1..n);

$\{1.500, 4., 5., 7., 8.500, 10., 11., 12.50\}$ $SX := 59.5000$

> SXX:=add(X[i]^2,i=1..n); $SXX := 541.750$

> {seq(Y[i],i=1..n)}: evalf(%,4); SY:=add(Y[i],i=1..n);

$\{4.500, 5., 6.500, 7., 9., 9.500, 11.\}$ $SY := 61.5000$

> SYY:=add(Y[i]^2,i=1..n); $SYY := 509.750$

> seq(Y[i]/X[i],i=1..n): evalf(%,4);

$3.333, 1.125, 1.400, 0.9286, 1.118, 0.9000, 1., 0.7200$

> SYX:=add(Y[i]/X[i],i=1..n); $SYX := 10.5246$

> seq(1/X[i],i=1..n): evalf(%,4);

$0.6667, 0.2500, 0.2000, 0.1429, 0.1176, 0.1000, 0.09091, 0.08000$

> S1X:=add(1/X[i],i=1..n); $S1X := 1.64809$

> seq(1/X[i]^2,i=1..n): evalf(%,4);

$0.4444, 0.06250, 0.04000, 0.02041, 0.01384, 0.01000, 0.008264, 0.006400$

> S1XX:=add((1/X[i]^2),i=1..n); $S1XX := 0.605857$

Determining the coefficients of the hyperbolic coupling equation (3):

> ab:=solve({a*S1X+n*b=SY,a*S1XX+b*S1X=SYX},{a,b});

$ab := \{a = -8.05421, b = 9.34676\}$

> yt:=ab[1]/x+ab[2]; $yt := \frac{a}{x} + b = -\frac{8.05421}{x} + 9.34676$

We set a and b in the system solution to y = a / x + b:

> a:=-8.05421; b:=9.34676; $a := -8.05421 \;\; b := 9.34676$

> Yt:={seq(a/X[i]+b,i=1..n)}: evalf(%,4);

{3.977, 7.333, 7.736, 8.196, 8.399, 8.541, 8.615, 8.702}

> seq((Y[i]-a/X[i]-b)^2,i=1..n)

1.04594, 8.02708, 0.541575, 2.87696, 1.21174, 0.210370, 5.69032, 0.0885521

> SYYt2:=add((Y[i]-a/X[i]-b)^2,i=1..n);

$SYYt2 := 19.6926$

> seq((abs((Y[i]-a/X[i]-b)/Y[i]))*1,i=1..n): evalf(%,4);

0.2045, 0.6296, 0.1051, 0.2609, 0.1159, 0.05096, 0.2169, 0.03306

> A:=seq(abs((Y[i]-a/X[i]-b)/Y[i]),i=1..8): A:=evalf(%,4);

$A :=$ 0.2045, 0.6296, 0.1051, 0.2609, 0.1159, 0.05096, 0.2169, 0.03306

> SA:=add(abs((Y[i]-a/(X[i])-b)/Y[i]),i=1..n); $SA := 1.61697$

> A100:={seq(abs((Y[i]-a/X[i]-b)/Y[i]),i=1..n)*100};

$A100 :=$ {3.30641, 5.09623, 10.5131, 11.5873, 20.4543, 21.6858, 26.0947, 62.9602}

> SA100:=add(abs((Y[i]-a/(X[i])-b)/Y[i]),i=1..n)*100;

$SA100 := 161.697$

> u:=array(1..10,1..10):

> for i to n do for j to n do

u[1,1]:="X[i]";u[1,2]:="Y[i]";u[1,3]:="X[i]^2";u[1,4]:="Y[i]/X[i]"; u[1,5]:="1/X[i]";

u[1,6]:="1/X[i]^2"; u[1,7]:="Yt=a/X[i]+b";u[1,8]:="(Y[i]-a/X[i]-b)^2"; u[1,9]:="A[i]";

u[1,10]:="A100";

u[i+1,1]:=X[i];u[i+1,2]:=Y[i];u[i+1,3]:=X[i]^2;u[i+1,4]:=Y[i]/X[i]; u[i+1,5]:=1/X[i]; u[i+1,6]:=1/X[i]^2;

u[i+1,7]:=a/X[i]+b; u[i+1,8]:=(Y[i]-a/X[i]-b)^2;

u[i+1,9]:=A[i];u[i+1,10]:=A100[i];

u[n+2,1]:=SX;u[n+2,2]:=SY; u[n+2,6]:=1/X[i]^2; u[n+2,3]:=SXX; u[n+2,4]:=SYX;

u[n+2,5]:=S1X;u[n+2,7]:=\`-\`; u[n+2,8]:=SYYt2; u[n+2,9]:=SA; u[n+2,10]:=SA100;

od;od; evalf(evalm(u),5);

"X[i]"	"Y[i]"	"X[i]^2"	"Y[i]/X[i]"	"1/X[i]"	"1/X[i]^2"	"Yt=a/X[i]+b"	"(Y[i]-a/X[i]-b)^2"	"A[i]"	"A100"
1.5000	5.	2.2500	3.3333	0.66667	0.44444	3.9773	1.0459	0.2045	3.3064
4.	4.5000	16.	1.1250	0.25000	0.062500	7.3332	8.0271	0.6296	5.0962
5.	7.	25.	1.4000	0.20000	0.040000	7.7359	0.54158	0.1051	10.513
7.	6.5000	49.	0.92857	0.14286	0.020408	8.1962	2.8770	0.2609	11.587
8.5000	9.5000	72.250	1.1176	0.11765	0.013841	8.3992	1.2117	0.1159	20.454
10.	9.	100.	0.90000	0.10000	0.010000	8.5413	0.21037	0.05096	21.686
11.	11.	121.	1.	0.090909	0.0082645	8.6146	5.6903	0.2169	26.095
12.500	9.	156.25	0.72000	0.080000	0.0064000	8.7024	0.088552	0.03306	62.960
59.500	61.500	541.75	10.525	1.6481	0.0064000	`-`	19.693	1.6170	161.70

From the values of these variables and the sum of the last line, we use the hyperbolic relationship in statistical estimates :.

1) The effectiveness of the found link:

> At:=add(abs((Y[k]-a/X[k]-b)/Y[k]),k=1..n)/n; $At := 0.202121$

> At:=At*100; $At := 20.2121$

Average deviation of values found in the experiment with regression correlation: 20.21%. This link-model is considered inefficient (a model-link calculation based on an approximation error of no more than 12-15% is a good model for forecasting).

2) Assess the adequacy of the connection: the number of n-tests.

> n:=8.: m:=1: k1:=n-1; k2:=n-m-1; $k1 := 7.$ $k2 := 6.$

General dispersion:

> S2y:=(SYY-SY^2/n)/k1; $S2y := 5.28129$

Residual dispersion:

> S2qol:=add((Y[k]-a/X[k]-b)^2,k=1..n)/k2; $S2qol := 3.28210$

F-Fisher criterion:

> F:=S2y/S2qol; $F := 1.60912$

According to Fisher's schedule $Ftab(\alpha,k1,k2)=Ftab(0.05,7,6)=4.21$

> Ftab:=4.21: #F<Ftab logorifimik bog'lanish adekvat

> F<Ftab:#(1.60<4.21)

3) Evaluation of the value of regression coefficients:

Find the random error of the coefficients:

> sigma[x]:=sqrt(SXX/n-(SX/n)^2); $\sigma_x := 3.52170$

> ma:=S2qol/(sigma[x]*sqrt(n)); $ma := 0.329499$

> mb:=S2qol*sqrt(SXX)/(n*sigma[x]) $mb := 2.71149$

We evaluate the value of the found regression coefficients with their random error according to the Student's criterion:

> #a:=.5325984252; b:=3.726299213;

> ta:=a/ma; $ta := -24.4438$

> tb:=b/mb; $tb := 3.44709$

From the Student Criteria Distribution Table (Appendix) we get the critical value: tkr=$t(\alpha,v)$=$t(\alpha,n$-2)=t(0.05, 6)=2.45

From this:

> tkr:=2.45:

> ta-tkr>0; $0 < -26.8938$ **# a statistik qiymtdor**

> tb-tkr>0; $0 < 0.99709$ **# b statistik qiymtdor emas**

From this we see that the coefficient a is not statistically significant and b is statistically significant. So, the hyperbolic relationship model is completely adequate to Fisher's F-criterion, and we found that some of its coefficients are not valuable. The model found can be used to draw conclusions and make decisions based on the results of a given experiment, but it cannot be used to make predictions.

We will now construct the graphs in a single coordinate system to compare the connections found.

Maple software

> restart;

> with(stats):with(plots):

> x1:=1.5:x2:=4:x3:=5:x4:=7:x5:=8.5:x6:=10:x7:=11:x8:=12.5:

> y1:=5:y2:=4.5:y3:=7:y4:=6.5:y5:=9.5: y6:=9:y7:=11:y8:=9:

> r1:=rhs(fit[leastsquare[[x,y],y=a*x+b,{a,b}]] ([[x1,x2,x3,x4,x5,x6,x7,x8],[y1,y2,y3,y4,y5,y6,y7,y8]]));

$r1 := 0.5325984252\,x + 3.726299213$

> r2:=rhs(fit[leastsquare[[x,y],y=a*ln(x)+b,{a,b}]]

([[x1,x2,x3,x4,x5,x6,x7,x8],[y1,y2,y3,y4,y5,y6,y7,y8]]));

$r2 := 2.704853634\ln(x) + 2.712806665$

> r3:=rhs(fit[leastsquare[[x,y],y=a/x+b,{a,b}]] ([[x1,x2,x3,x4,x5,x6,x7,x8],[y1,y2,y3,y4,y5,y6,y7,y8]]));

$$r3 := -\frac{8.053943729}{x} + 9.346692907$$

> NXY:=pointplot([[x1,y1],[x2,y2],[x3,y3],[x4,y4],[x5,y5],

[x6,y6],[x7,y7],[x8,y8]],thickness=3,symbol=BOX,symbolsize=30,color=red):

> p1:=plot(r1,x=1..16,4..13,linestyle = [solid], legend=["y=ax+b"],thickness=3,color=[blue]):

> p2:=plot(r2,x=1..16,4..13,linestyle = [longdash], legend=["y=aln(x)+b"],thickness=3,color=[red]):

> p3:=plot(r3,x=1..16,4..13,linestyle = [dashdot], legend=["y=a/x+b"],thickness=3,color=[black]):

> display(p1,p2,p3,NXY); (Figure 1)

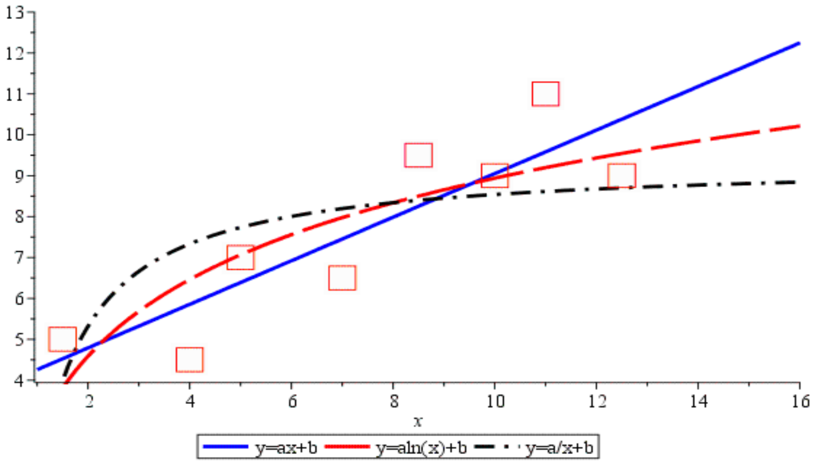

Figure 1. Link comparison chart.

Regression equation	Degree of freedom		Dispersion		Fisher's criterion		The average error aproc.	Student Criteria		
	k_1	k_2	general	residue	F_{tab}	$F_{k1,k2}$	A_i, 100	t_1	t_2	t_{kr}
y=0.53x+3.73	7	6	5.28	1.47	3.59	4.21	13.11	3.06	3.6	2.45
y=2.71·ln(x)+2.7	7	6	5.28	2.00	2.64	4.21	15.58	1.64	1 3.57	2.45
y=-8.05/x+9.34	7	8	5.28	3.28	3.28	4.21	20.21	3.44	-24.4	2.45

CONCLUSION. Thus, we can see from the last table that the linear link-model found on the basis of the small squares method is the best regression equation for the experimental results: the smallest residual variance ($\overline{S}^2_{qold} = 1.47$), the largest Fisher criterion F (F = 3.59) and the mean of the data to the smallest deviation-approximation (13.11%), both coefficients are valuable according to the Student's criterion ($t_1 > t_{kr}$, $t_1 > t_{kr}$). This means that the linear relationship formed for the experimental results is completely adequate, and we found that all its coefficients are valuable. We have seen that accurate, fast, and high-quality results can be obtained using Maple to build a model and determine its effectiveness to draw conclusions and make decisions based on the results of a given experiment.

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