General Statement of the Problem of Bank Sustainability Management and Methods for its Solution

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Annotation:
A stable banking system is a critical link in the development and successful operation of a market economy, as well as a vital requirement for its growth and stability in general. With its constant complication, improved banking legislation, increased competition in the banking environment, and low profit margins, banks are faced with the need to choose a development strategy that links the various priorities of their activities and the various interests of all economic entities, one way or another, interested in bank business into a single whole. In this thesis we aimed to develop models for managing a bank's financial stability using multi-criteria optimization techniques.

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Literature review. Theoretical problems associated with the formation of a bank development strategy and its impact on its profitability and liquidity were considered in the works of E. Reid, R. Kotter, E.J. Dolan, J. Sinki, O.I. Lavrushina, V.M. Usoskin, M.B. Dichenko, Z.T. Tomaeva and other scientists. These works are of great theoretical and practical importance. However, until now, scientific views on the problem of forming a bank development strategy that ensures a balance of interests of all economic entities interested in its activities are not sufficiently systematized, resulting in the lack of a systematic approach to solving the problem of "profitability or liquidity" in relation to banking.

Mathematical methods for modeling banking activity were considered by F. Edgeworth, E. Baltensperger, K. Seeley, M. Klin, B.C. Kromonov, O.I. Katugin, A.V. Buzdalin and other researchers. Their works reflect many aspects of the creation and use in practice of economic and mathematical models of banking activities, the formation of ratings for evaluating the reliability of banks. However, until now, when formalizing the methods of managing the assets and liabilities of a bank, most researchers prefer to build models that operate with scalar criteria for optimizing its activities, using which only one studied indicator is considered the most important characteristic of the functioning of a credit institution. At the same time, due to the specific features of the mathematical tools used, a solution can be obtained in which the remaining indicators are brought to the maximum allowable boundaries of their change intervals, which in practice leads to an insufficiently flexible solution to the "profitability - liquidity" dilemma.
Materials and methods. The task of managing the stability of a commercial bank in its economic essence is multi-criteria. In general terms, it can be formulated as follows. Let \( L \) criteria for the quality of the bank's work be given:

\[
P_i = f_i(\bar{x}), \ i = 1, 2, 3, \ldots, L, \ \bar{x} \in X,
\]

where \( \bar{x} = (x_1, x_2, \ldots x_n) \) is a vector of variable parameters-investments of the bank in various assets that generate income;

\[
X = \{ x | g_j(\bar{x}) \geq 0, j = 1, 2, \ldots, m \} - \text{admissible closed set of variable parameters;}
\]

\[
f_j, g_j - \text{some features } \bar{x}.
\]

Then the optimization problem is reduced to determining the vector of optimal parameters \( \bar{x}_0 \in X \) such that

\[
P_i(\bar{x}_0) = \text{opt} P_i(\bar{x}) \ i = 1, 2, \ldots, L
\]

Problem (1.3.1) is an optimization problem with respect to the vector criterion

\[
\vec{P} = (P_1, P_2, \ldots, P_L),
\]

which is characterized by the uncertainty of goals, i.e. the impossibility in most cases to simultaneously maximize (minimize) all components of the vector criterion [1].

As is known, in contrast to the problem of scalar optimization, the problem of optimization by several criteria in the general case does not have a trivial solution. Therefore, the decisive factor in choosing one or another solution technique is a thorough analysis of the economic meaning of the criteria, their relative importance, a clear understanding of the qualitative characteristics reflected by quantitative criteria.

Therefore, it is necessary to analyze the main existing methods for solving problems of multicriteria optimization and evaluate their applicability [2-3].

Depending on the relative importance of the criteria, the following optimization methods are used, which we classify in terms of the forms and degree of participation of the decision maker (DM), into interactive ones, during the application of which there is a constant dialogue with the decision maker, and fully automatic, in which the preferences of the decision maker are set once at the beginning of the solution, and then the technique gives out one point as the optimal one: the methods of the main criterion, lexicographic optimization, convolutions, concessions, construction of the Pareto-optimal set with subsequent expert choice (for a classic review of these methods, see R. Steuer [4]).

Method of the main criterion. The simplest and most frequently used method is to single out one criterion as the main one and transfer the remaining criteria to the category of restrictions by formulating additional restrictions on the values of these criteria. This method is applicable in the case when one of the criteria reflects the main goal of the object functioning, and the rest are some auxiliary goals [5, 6, 7]. The advantages of clearness, ease of interpretation of the results and low requirements for the mathematical training of the expert, software and computer speed have led to the widespread use of this method in a variety of models of bank balance optimization. The selection of the main criterion should be preceded by the procedure for constructing an adequate current economic situation of the system of criteria. For this, A.V. Buzdalin [8-9] suggested using nonparametric statistics methods. The application of his method requires an initial classification of banks into "reliable" and "unreliable". Expert assessments, information about bankruptcies and cases of delayed payments, etc. can be used as such a classification. The values of balance sheet accounts, their relationship to the total amount of assets, profits, equity capital, the values of the Bank of Russia norms and others can be used as numerical indicators of the activities of banks. Numerical indicators are called individually significant if their change leads to a change in the financial stability of banks when it is impossible to compensate for a negative change in one characteristic by a positive change in another (since the standards should signal financial instability even when one of them goes beyond threshold values, while others do not come out). To identify significant characteristics and their
values, it is possible to use the methods of parametric and nonparametric statistics. At the first stage, the
widest possible list of bank characteristics available for analysis is created, based on the available data, a
sample is created from the values of the analyzed characteristic, after which, according to the existing
classification of banks into "reliable" and "unreliable", the resulting sample is divided into two
\((x_{1j}^1, x_{2j}^1, ..., x_{nj}^1)\) where j=1,2 for reliable and unreliable banks, respectively. In the case of the significance of
the corresponding characteristic, these samples should have different statistical parameters, that is, they are
heterogeneous (having different probability distribution laws). To test the hypothesis of distribution
homogeneity, the Kolmogorov-Smirnov test should be used, based on a comparison of the empirical
distribution functions of samples that characterize the data distribution laws in general terms. For samples of
stable and unstable banks, the empirical distribution functions will take the form:

\[
F_j(z) = \frac{1}{n_j} \sum_{m=1}^{n_j} L(x_{mj}^j \leq z) j = 1,2 (2)
\]

Where \(L(x_{mj}^j \leq z)\) - function that takes the value 1, if \((x_{mj}^j \leq z)\) and 0 otherwise (z is an argument that
changes with a certain step). Then the required value \(T\), characterizing the degree of homogeneity
(similarity) of the samples, will be determined by the expression:

\[
T = \sqrt{\frac{n_1n_2}{n_1+n_2}} \max |F_1(z) - F_2(z)| ,
\]

(3)

where \(n_1\), \(n_2\) -the number of banks in the groups of solvent and insolvent capable.

The closer \(T\) is to 0, the more homogeneous the samples, and the more it differs from 0, the less identical the
samples. It is recommended to take \(T = 1.22\) as the critical value of \(T\), above which it is reasonable to
consider the samples to be heterogeneous, and the characteristic to be significant. Thus, at the first stage, out
of the entire set of characteristics, those whose samples in the groups of reliable and unreliable banks are not
identical (7>1.22) are selected as significant. At the second stage, it is necessary to evaluate the threshold
values of significant characteristics of the bank's work, that is, to identify the areas of their allowable
changes.

As a rule, the area of acceptable changes is given by a number, such that if the value of the characteristic lies
above (below) this number, then the probability of a successful state of the corresponding bank is higher
than that of an unfavorable one, and vice versa. This principle is formalized in statistics using the
classification method based on the "likelihood ratio". Based on the analysis of empirical distribution
functions, a new special function is constructed equal to their difference:

\[
G(z)=F_1(z)-F_2(z). (4)
\]

Next, a graph of this function is constructed, smoothed in one way or another (for example, by the moving
average method), and areas of monotonous growth and decline are clearly separated on it. In this case, the
area of monotonic growth is the area of acceptable values of the characteristic, and the area of monotonic
decline is unacceptable.

The lexicographic optimization method [10, 11] is used when the criteria are clearly ranked by priority, and
each next criterion is absolutely less important than the previous one, that is, the concession on the first
criterion is not compensated by any increment on the other. This method reduces the solution of a multi-
criteria problem to a series of single-criterion ones, when the first criterion is optimized first, then the
second, provided that the value of the first remains maximum, and so on. The impossibility of applying the
lexicographic optimization method in solving the problem of optimal asset management is confirmed by the
complete absence of proposals for its application in this area.

In contrast, criteria convolution is a very common group of methods for scalarization of a vector
mathematical programming problem, often proposed in asset optimization problems. There are a large
number of different types of convolutions (Yu.K. Mashunin [12]). Theoretically, all of them are based on the concept of the decision maker's utility function (R. Steuer [4], P. Fishburne [13]). With this approach, it is assumed that the decision maker always has a utility function, regardless of whether it is possible to specify it explicitly (that is, to give its mathematical description). This function maps the criteria vectors onto the real line so that the larger value on this line corresponds to the more preferred criteria vector. The meaning of different convolutions is to obtain one summary criterion from several criteria, thus approximately modeling the unknown (not explicitly specified) function usefulness of the decision maker. The most popular convolution is the method of weighted sums with point estimation of the weights. In this case, a vector of criteria weight coefficients is set, which characterizes the relative importance of a particular criterion.

\[ A = \{a_k \mid k = 1, 2, \ldots, K\} \]  
(5)

Where, \(a_k\) = weight coefficients;

K is the total number of criteria.

Weight coefficients are usually used in a normalized form and satisfy the following conditions:

\[ \sum_{k=1}^{K} a_k = 1 \]  
(6)

\[ a_k \geq 0 \mid k = 1, 2, \ldots, K \]

those, the weights are assumed to be non-negative and their sum equals 1. Each criterion is multiplied by its own weight, and then all the weighted criteria are summed to form a weighted objective function. The resulting scalarized function is maximized on the allowable constraint area, and a single-objective (scalar) mathematical programming problem is obtained:

\[ F^0 = \max_x \sum_{k=1}^{K} a_k f_k(x) \]  
(7)

As a result of solving this problem, the optimum point \(x^0\) is calculated.

**Conclusion.** However, this method has a number of fundamental drawbacks [14, 15]. First, the decision maker's implicit utility function is generally non-linear, so the "true" weights of the criteria (i.e., weights such that the gradient of the weighted objective function coincides in direction with the gradient of the utility function) will vary from point to point, therefore, we can only speak of locally appropriate weights, moreover, often the decision maker cannot specify weight coefficients at all. Secondly, the loss of quality according to one of the criteria is not always compensated by the increase in quality according to another. Therefore, the resulting solution, which is optimal in the sense of a single summary criterion, may be characterized by low quality in terms of a number of particular criteria and, therefore, be absolutely unacceptable. Thirdly, the convolution of criteria of different physical nature does not allow us to interpret the value of the weighted objective function. Some of the above disadvantages can be adjusted. So, in the case of different physical (economic) nature of the criteria, their normalization and subsequent convolution of the normalized criteria are possible. To exclude unacceptably low values of individual criteria, you can impose additional restrictions on these criteria.

**References**

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