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DIFFERENTIAL SCHEMES FOR THE VIBRATION EQUATION OF A STRING

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Abstract

In this article, a differential and differential problem for the vibration equation of a string belonging to the class of hyperbolic type equations was set, and a calculation algorithm was derived.

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Introduction

Setting the differential problem. Consider the equation of motion of a homogeneous string.

$$\frac{\partial^2 U}{\partial t_1^2} = a^2 \frac{\partial^2 U}{\partial x_1^2} + f(x_1; t_1),$$

$$0 < x_1 < l \; t_1 > 0$$

We write this equation in this form by introducing dimensionless variables $x = x_1/l, t = at_1/l$.

$$\frac{\partial^2 U}{\partial t^2} = a^2 \frac{\partial^2 U}{\partial x^2} + f(x,t), (1)$$

$$0 < x < 1, 0 < t \le T \;,$$

The following conditions are given at the initial moment of time (t=0).

$$\begin{cases} U(x;0) = U_0(x) \\ \frac{\partial U(x;0)}{\partial t} = \overline{U}_0(0) \end{cases} \qquad (2)$$

Here, $U_0(x)$ is the initial deviation of the string, and $\bar{U}_0\ (x)$ is the initial speed of the string. And the edge points of the string move according to the given law

$$\begin{cases} U(0;t) = \mu_1(t) \\ U(1;t) = \mu_2(t) \end{cases}$$

Setting a separate issue.. In the given area $\overline{D} = \{0 \le x \le 1, 0 \le t \le T\}$, we introduce the differential string $\overline{\omega_{h\tau}}$.

$$\overline{\omega_{h\tau}} = \begin{Bmatrix} (x_i\,; t_j), x_i = ih, i = 0,1,2 \dots H, h = {}^1\!/_H \\ t_j = j\tau, j = 0,1,2 \dots M, \tau = {}^1\!/_M \end{Bmatrix}$$

Since equation (1) has a second derivative with respect to time t, the number of layers of the network should not be less than three.

We use the following notation.

$$y = y^j, \hat{y} = y^{j+1}, \check{y} = y^{j-1},$$

$$y_t = (\hat{y} - y)/\tau\,; y_{\bar{t}} = \frac{y-\hat{y}}{\tau}, \Lambda y = y_{\bar{x}x},$$

$$y_{\bar{t}t} = \frac{y_t - y_{\bar{t}}}{\tau} = \frac{\hat{y} - 2y + \check{y}}{\tau^2},$$

$$y_{t^0} = \frac{y_t - y_{\bar{t}}}{2} = \frac{\hat{y} - \check{y}}{2\tau}$$

We replace the partial derivatives in equation (1) with the following differential derivatives.

$\frac{\partial^2 U}{\partial t^2} \sim U_{\bar{t}t}\,, \frac{\partial^2 U}{\partial x^2} \sim \Lambda U = U_{\bar{x}x}\,, f \sim \varphi$

Let's look at the weighted scheme below.

$$\begin{cases} y_{\bar{t}t} = \Lambda(\delta\hat{y} + (1 - 2\delta)y + \delta\,\check{y}) + \varphi, \\ \varphi = f\left(x; t_j\right), \\ y_0^{j+1} = \mu_1\left(t_{j+1}\right), \\ y_M^{j+1} = \mu_2\left(t_{j+1}\right) \\ y(x;0) = u_0(x) \\ y_t(x;0) = \hat{u}_0(x) \end{cases} \tag{4}$$

In this case, we will determine $\hat{U}_0(x)$ later, δ - weight parameters, real number. The boundary conditions and the first initial condition of the differential scheme (4) are clearly satisfied on the $\varpi_{h\tau}$ grid. Now we choose $\hat{U}_0(x)$ so that the approximation error $\hat{U}(x) - \partial u(x;0)/\partial t = \hat{U}(x) - \overline{U}(x)$ is a quantity of order $O(\tau^2)$ for this expression.

From the following

$$U_t(x;0) = U'(x;0) + 0{,}5\tau U''(x;0) + O(\tau^2) =$$
$$= \overline{U}_0(x) + 0{,}5\tau\left(U''(x;0) + f(x;0) + O(\tau^2)\right) =$$
$$\overline{U}_0(x) + 0{,}5\tau\left(U_0''(x) + f(x;0) + O(\tau^2)\right)$$

It can be seen that condition

$$\widehat{U}(x) - U_t(x;0) = 0\left(\tau^2\right)$$

is fulfilled, if

$$\widehat{U}(x) = \overline{U}_0(x) + 0.5\tau\left(U_0''(x) + f(x;0) \right) \quad (5)$$

Thus, separate issue (4)-(5) was set. Now we come to the following boundary value problem to determine $\hat{y} = y^{j+1}$ from (4).

$$\delta J^2 (y_{i-1}^{j} + y_{i-1}^{j+1}) - (1 + 2\delta J^2) y_i^{j+1} = F_i$$

$$0 < i < N\ ,\ y_0 = \mu_1\ ,\ y_N = \mu_2\ ,\ J = \tau / h$$

$$F_i = (2y_i^{j-1} - y_{i+1}^{j-1}) + \tau^2 (1 - 2\delta) \wedge y_j + \delta t^2 \wedge y^{j-1} + \tau^2 \varphi$$

This differential scheme is solved by the progonka method. The progonka method is stable at $\delta > 0$.

Approximation error. Let's check the approximation error of scheme (4) in case of $\varphi = f(x;t_j)$. Suppose that y is the solution of the differential problem (4)-(5), and $U = U(x;t)$ is the solution of the differential problem (1)-(3).

Now, putting y = z + u in (4), we get the following.

$$z_{\bar{t}t} = \wedge(\delta \hat{z} + (1 + 2\delta) z + \delta \check{z}) + \varphi \quad (6)$$

$$z_0 = z_N = 0$$

$$z(x;0) = 0$$

$$z_t(x;0) = \partial(x)$$

Here $\psi = \wedge(\delta \widehat{U} + (1 - 2\delta)U + \delta \widehat{U}) + \varphi - U_{\bar{t}t} \rightarrow$ the solution of the differential problem $U(x;t)$ is the approximation error

$\partial = \widehat{U}_0(x) - U_t(x;0)$ of the differential problem (4) - approximation error for the second initial condition $y_t = \widehat{U}_0(x)$.

Taking into account the following $\widehat{U} = U - \tau U_t$ $\breve{U} = U - \tau U_t$ we get

$$\delta \widehat{U} + (1 - 2\delta) U + \delta \breve{U} = U + \delta t^2 U_{\bar{t}t},$$

that is,

$$\psi = \Lambda U + \delta \tau^2 \Lambda U_{\bar{t}t} + \varphi - U_{\bar{t}t} = LU + \delta \tau^2 L\ddot{U} + f - \ddot{U} + 0(\tau^2 + h^2) \quad (7)$$

Thus, the approximation error of the differential scheme (4) is equal to $\psi = 0(\tau^2 + h^2)$ for arbitrary δ (parameter δ does not depend on grid steps τ and).

If there is $\delta = \bar{\delta} - h^2/(12\tau^2)$, then $\bar{\delta}$ parameter (independent of $y\tau$ and h) is selected under the condition that scheme (4) is stationary. In this case, it is enough to require $\bar{\delta} \geq 1/(4(1-\varepsilon))$, because in this case scheme (4) is stable.

In this case, it is enough to require $\bar{\delta} \geq 1/(4(1-\varepsilon))$, because in this case scheme (4) is stable.

$$\bar{\delta} \geq 1/(4(1-\varepsilon)) - 1/(4J^2) \quad J = \tau / h, \ \varepsilon > 0$$

If so

$$\varphi = f + \frac{h^2}{12} f'' \qquad (8),$$

the scheme (4) has a high-order approximation, that is, $\psi = 0(\tau^2 + h^4)$.

Example 1. Solve the $\frac{\partial^2 U}{\partial t^2} = a^2 \frac{\partial^2 U}{\partial x^2}$ equation with $a = 4*10^2 m/s$ $l=4m$ and the following initial and boundary conditions. (using the finite difference method)

$$U(x;0) = U_0(x) = \begin{cases} x, if, x \in [0;1] \\ \frac{4-x}{3}; if, x \in [1;4] \end{cases} \qquad \text{(a)}$$

$$\frac{\partial U(x;0)}{\partial t} = \overline{U}_0(x) = \begin{cases} 10x, if, x \in [0;1] \\ 10, if, x \in [2;3.2] \\ 12.5(4-x), if, x \in [3.2;4] \end{cases} \qquad \text{(b)}$$

$$U(0,t) = 0 \quad U(l,t) = 0 \qquad \text{(c)}$$

Solving. The issue

$$y_i^{j-1} + \gamma^2(y_{i-1}^j + y_{i-1}^j) + 2(1-\gamma^2)y_i^j + \tau^2\varphi_i^j \quad i = \overline{1, I-1} \quad \text{(d)}$$

we solve using the open finite difference method with $\varphi_i^j = 0$.

We choose the step $h=0.4m$ according to the x_i coordinate. In this case, we choose the step for $I=10$. τ from the Courant condition.

$$\left(\tau \le \frac{h}{a}\right) \quad \tau \le \frac{0.4}{4*10^{2}} = 10^{-3}s$$

As a result, $\tau = 0.8*10^{-3}s$ can be obtained. For these values of *a, τ, h,* we have the following.

$$\gamma^{2} = \frac{a^{2}\tau^{2}}{h^{2}} = \frac{16*10^{4}*0.64*10^{-6}}{0.4^{2}} = 0.64$$

In the zeroth layer, the solution is defined by the $U_0(x)$ function, and in the first layer, $\overline{U}_0(x)$, $U_0(x)$ is a linear function, and $f(x;t) = 0$, so there is no need to apply condition (5). (We do not pay attention to the fact that there is no $U_0''(x)$ derivative at the point x=1).

Using (a), we get the following.

$$y_0^0 = U_0(0) = 0$$

$$y_1^0 = U_0(0.4) = 0.4$$

$$y_2^0 = U_0(0.8) = 0.8$$

…………………………………..

$$y_{10}^0 = U_0(4) = (4-4)/3 = 0$$

If we use $y_i^1 = U_0(x_i) + \tau\overline{U}_0(x_i)$ in the first layer,

$$y_0^1 = U_0(0) + \tau\overline{U}_0(0) = 0$$

$$y_1^1 = U_0(0.4) + \tau\overline{U}_0(0.4) = 0.4 + 0.008*4 = 0.4032$$

$$y_2^1 = U_0(0.8) + \tau\overline{U}_0(0.8) = 0.8 + 0.0008*8 = 0.8064$$

$$y_3^1 = U_0(0.9333) + \tau\overline{U}_0(0.9333) = 0.9333 + 0.000093*16 = 0.9413$$

……………………………………………………………………………

$$y_{10}^1 = 0$$

When $\gamma^{2} = 0.64$ from (d), the following expression is formed.

$$y_i^{j-1} = -y_i^{j-1} + 0.64(y_{i-1}^j + y_{i-1}^j) + 0.72 y_i^j \quad i = \overline{1,9}$$

In particular, when *j=1*:

$$y_1^2 = -y_1^0 + 0.64(y_0^1 + y_2^1) + 0.72 y_1^1 =$$

$$-0.4 + 0.64(0 + 0.8064) + 0.72 * 0.4032 = 0.4064$$

$$y_2^2 = -0.8 + 0.64(0.4032 + 0.9413) + 0.72 * 0.8064 = 0.8118$$

....

$$y_9^2 = -0.1333 + 0.64(0.2747) + 0.72 * 0.1373 = 0.1414$$

$$y_{10}^2 = 0$$

In the same way, solutions are determined in layers *j=3,4,…* . Solutions in several layers are given in Table 1.

I	x_i^1	y_i^0	y_i^1	y_i^2	y_i^3
0	0.0	0.0	0.0	0.4	0.0
1	0.4	0.4	0.4032	0.4064	0.409
2	0.8	0.8	0.8064	0.8118	0.5359
3	1.2	0.9333	0.9413	0.7776	0.6604
4	1.6	0.8	0.808	0.816	0.714
5	2.0	0.6667	0.6747	0.6826	0.6906
6	2.4	0.5333	0.5413	0.5494	0.5574
7	2.8	0.4	0.408	0.416	0.4224
8	3.2	0.2667	0.2747	0.2801	0.2837
9	3.6	0.1333	0.1373	0.1414	0.1438
10	4.0	0.0	0.0	0.0	0.0

Table 1

As can be seen from Table 1, the wave amplitude around $x_3 = 1{,}2m$ is decreasing from the second time layer. This decrease occurs on both sides of the point.

Conclusion

The search for the solution of differential equations is widely used, along with analytical methods, using various methods. The main reason for their wide development is due to the emergence of modern computers, the possibility to store very large tables of numbers in memory and perform arithmetic and logical operations based on the given program.

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