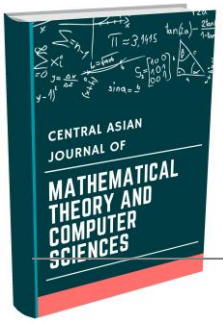

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New Discrete Metric Space of Natural Numbers: Discovery of New Possibilities in Number Theory

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Abstract

In this article, the main emphasis is on the importance of the recently discovered by the author of the first in the history of mathematics metric criterion to be prime for a natural numbers. In his recent article, the author discovered a new discrete metric space, where, as it turned out quite unexpectedly, the distance between points is exactly two times less than the Hamming distance between the corresponding binary vectors. Extending this new distance in a natural way to the set of natural numbers, the author formulated a truly remarkable metric criterion for natural numbers to be prime.

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First, let us state (this time without proof) the main propositions published (with proofs) by the author recently in [2].

For any finite set **A**, through **|A|** we denote the number of elements of this set, that is, in the language of set theory – the cardinality of this set. For example, if **A = {4, a, 7, b},** then **|A| = 4**.

Now let **T** be an arbitrary (finite or infinite) set, each element of which is a finite set. In other words, **T** is the set of some finite sets. Here the word "some" has a very broad meaning: the set **T** can consist of an infinite number of such "some" (that is, arbitrary, any) finite sets. These finite sets contained in **T** can be very different: heterogeneous or homogeneous. Despite this, in the most ordinary, generally accepted sense, we will use the operations of union and intersection of these finite sets, and these unions and intersections do not have to be elements of the set **T**.

For any elements **α ∈T** and **β ∈T**, we introduce the distance **ρ (α, β)** between them by the following formula:

$$\rho(\alpha,\beta) = ((|\alpha| + |\beta|))/2 - |\alpha \cap \beta|, \qquad (1)$$

here **α∩β** is the intersection of the subsets **α** and **β**.

For example, if **T** contains elements **α = {1, 2, 3}** and **β = {3, 4, 5, 6}**, then using formula **(1)** it is easy to calculate the distance between them. Because

| α | = 3, | **β** | **= 4** and | **α∩β** | **= 1**,

then

ρ (α, β) = ((3 + 4))/2 – 1 = 3.5 – 1 = 2.5.

The following theorem holds.

Theorem 1. The set T with distance (1) is a metric space.

Consider an arbitrary finite set **X** consisting of **n** elements. Let these elements be numbered, i.e.

$\mathbf{X = \{x_1, x_2, x_3, \ldots, x_n\}}$.

Now let **T** be the set of some subsets of the set **X**. Then, according to **Theorem 1** just proved, the set **T** is a metric space with metric **(1)**. For any **α ∈T** and **β ∈T**, we define binary vectors of length **n**:

$$\overline{\alpha} = (a_1, a_2, \ldots, a_n), \overline{\beta} = (b_1, b_2, \ldots, b_n),$$

where

$$a_i = \begin{cases} 1, \text{if } x_i \in \alpha, \\ 0, \text{if } x_i \notin \alpha; \end{cases}$$
$$b_i = \begin{cases} 1, \text{if } x_i \in \beta, \\ 0, \text{if } x_i \notin \beta; \end{cases}$$
$i = 1, 2, 3, \ldots, n$.

As is known, the Hamming distance $h(\overline{\alpha}, \overline{\beta})$ (see [1, p.39]) between these vectors is the number of their coordinates that differ in value.

It turns out that the new distance **(1)** we introduced between the sets is exactly two times less than the Hamming distance between the corresponding binary vectors. The following theorem holds.

Theorem 2. The equality

$$h(\overline{\alpha}, \overline{\beta}) = 2\rho(\alpha, \beta). \qquad (2)$$

Thus, the new distance **(1)** introduced by us is exactly two times less than the corresponding Hamming distance in the case when the set **T** is the set of subsets of some finite set. Therefore, to calculate the Hamming distance between two binary vectors, it is enough to first calculate the distance **(1)** between the corresponding subsets, and then multiply it by two. To calculate the distance **(1)** is somewhat easier, because, as follows from the proof of **Theorem 2**, only those coordinates of the two binary vectors being compared are considered, where at least one vector has a coordinate value of **1**. And those coordinates where both binary vectors have zero values are not considered (and therefore not compared). This means that the number of comparisons when calculating the Hamming distance by formula **(2)** decreases. This means that calculating the new distance **(1)** is easier than calculating the corresponding Hamming distance.

It should be emphasized that the distance **(1)** introduced by us is in a certain sense ***universal*** than the Hamming distance, since only one condition is imposed on the set **T**: it is only required that the set **T** be the set of some (any) finite sets; and these finite sets can be finite subsets of ***any***, including infinite, sets.

Now, as ***an application***, consider the set **N** of natural numbers and transform it into a completely new (unusual, previously unexplored) metric space using distance **(1)** as follows. To each natural number **n** we associate the set n_d of all its divisors. It is clear that n_d is a finite set for any **n**. Now we introduce the distance $\eta(a,b)$ between two natural numbers ***a*** and ***b*** by the formula

$$\eta(a,b) = \rho(a_d, b_d), \qquad (3)$$

where $\rho(a_d, b_d)$ is the distance **(1)**. Then, according to **Theorem 1**, the set **N** of natural numbers will be a metric space N_η with distance **(3)**. This new infinite discrete metric space N_η can be the subject of close study from the point of view of classical number theory. But this may already be a topic for further research. It should be noted that in the space N_η the distance between any two adjacent powers of a prime is always $\frac{1}{2}$. Moreover, the following theorem holds.

Theorem 3. A natural number $\boldsymbol{p} \in \boldsymbol{N}$ is prime if and only if for any natural number $\boldsymbol{m} \in \boldsymbol{N}$, the equality $\boldsymbol{\eta(p^m, p^{m+1}) = \frac{1}{2}}$.

In other words, a natural number is prime if and only if the distance between its neighboring powers is $\frac{1}{2}$. A complete exhaustive proof of this theorem is given in [2]. This theorem expresses a truly remarkable, completely new criterion for natural numbers to be prime. These are only the first results of the new discrete metric space. It is quite possible that many unsolved problems of number theory can be solved easily and simply in this new discrete metric space of natural numbers, and the existing cumbersome solutions of known problems can be substantially simplified. The research is ongoing. The author gladly agrees to cooperate with leading scientists of the world. You can contact me by e-mail: jamolidinkamol@gmail.com.

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