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COMPARISONS OF MODERN TURBULENCE MODELS FOR THE POISEUILLE-COUETTE-TAYLOR PROBLEM

Makhsitalayev Barhayot Iftikharjan ugli, Rustamova Ku.O.
Ferghana Polytechnic Institute, E-mail:Madaliev.ME2019@mail.ru, m.e.madaliyev@ferpi.uz

Annotation

The article investigates the turbulent flow of a fluid flow in a rotating pipe, which is called the Poiseuille-Couette-Taylor flow. The main approaches to the numerical simulation of turbulent flows in an annular region between rotating cylinders are considered. Calculated results are obtained that correlate with known experimental results. On the basis of a comparative analysis, the most appropriate differential turbulence model for calculating the coupled problems of hydrodynamics and heat transfer in a Poiseuille-Couette-Taylor flow is proposed.

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Turbulence modeling currently used in aerodynamics is usually based on two-equation models using a linear relationship between Reynolds stress and mean strain rate tensors. This relationship is known as the generalized Boussinesq model. This can be too limiting an assumption in complex problems typical of high lift aerodynamics because many different flow phenomena can be present in the same problem. Therefore, it is necessary to look for turbulence modeling with a wider range of applicability than the Boussinesq models. Reynolds differential stress modeling (RSM), in which a simulated transport equation is solved for each stress component, is in principle a more general class of models with a wider range of applicability. However, RSM is considered too complex an industrial design approach for high lift aerodynamics. On the other hand, 2-equation models can be extended to a wider range of applicability by developing more complex non-linear relationships between stress tensor and mean velocity gradient and turbulent scales. These relationships are commonly referred to as constitutive patterns. One might think that two-equation turbulence models consist of two more or less separate parts: a scaling model that provides scalar information about the turbulence, and a constitutive model that defines the Reynolds stress tensor. Explicit algebraic Reynolds stress models (EARSM) are an interesting and promising subset of non-linear constitutive models. In this approach, part of the description of higher-order physical processes at the RSM level is transferred to the level of modeling with two equations. The EARSM approach is considered to be the appropriate type of constitutive modeling for the present purposes.

In the world, the most interesting and frequent currents are turbulent. Our life is closely connected with them, which provides a significant reason for their careful study. For many years, the compilation of a mathematical model of turbulent flows has been a difficult task. And although turbulent flows obey the Navier-Stokes equations, the problem does not have exact solutions. This is due to the fact that computer technology is not able to sufficiently solve these equations. It follows from this that it is necessary to look for a certain compromise between the correspondence of the physical model and the level of computer technology.

There are several basic approaches to modeling turbulent flows. Direct numerical simulation(Direct Numerical Simulation). The method is based on the solution of non-stationary Navier-Stokes equations, where spatial grids and integration steps in time are used. This method requires the use of very fine meshes. The method is used for simple turbulent flows [1].

Method of large eddy simulation (Large Eddy Simulation). The essence of the method is that the Navier-Stokes equations are “filtered” only from short-wave turbulent in homogeneities. Due to this, consideration of part of the scales is excluded. This gives a great advantage in DNS comparisons, as the requirements for the necessary computing resources are reduced [2].

Application of Reynolds-averaged Navier-Stokes equations closed by turbulence models (Reynolds Averaged Navier-Stokes). In RANS a model, all turbulent eddies are modeled [3].

Integral methods. Methods combine a large amount of experimental data to determine the classes of flows. These are strong methods for making industrial calculations. They have a big drawback - the method is not universal [4].

Simulation of detached eddies, hybrid method (DetachedEddySimulation). An approach that combines LES and RANS has been widely adopted. In the boundary layer reattachment region, DES performs its function in the Reynolds equation mode. In the separation area, it switches to the LES method. There is a combination of the best qualities of these methods [5].

Recently developed a new approach to turbulence based on the dynamics of two fluids [6]. The essence of this approach lies in the fact that in this work it is shown that a turbulent flow can be represented as a mixture of two heterogeneous fluids with different velocities. The peculiarity of the new model is that it is capable of describing complex anisotropic turbulent flows and, meanwhile, it is simple and economical for solving practical hydrodynamic problems.

The study of turbulent swirling liquid and gas flows is of great importance in connection with their wide distribution in nature and the use of technology. In such flows, the effect of rotation on turbulence significantly changes the characteristics of the turbulent transfer of momentum, heat, and mass. The ability of concentrated vortex formations to move in the environment over long distances to carry various kinds of impurities is determined, in particular, by the strong suppression of radial turbulent diffusion under the influence of centrifugal force caused by rotation. Flow swirling is used in a large number of practical applications, for example, to stabilize and intensify combustion processes in combustion chambers of various types of power plants.

The nature of the swirl effect on the flow structure is also determined by the way in which vorticity is created in the flow. Of all the variety of ways to impose vorticity on a turbulent flow, this paper considers the method of a rotating pipe: the rotation of the flow is created by the walls of a straight round pipe that rotates about the longitudinal axis. Swirling flows created by the pipe rotation method are used in various technical devices, for example, in the inlet part of hydraulic machines, heat exchangers and rotor cooling systems.

As a rule, experimental modeling of swirling turbulent flows, including those generated by the pipe rotation method, is difficult both from a technical and economic point of view, therefore, mathematical modeling of turbulent transfer processes in swirling flows is the most effective means of obtaining reliable data.

The strong anisotropy of turbulent transport in swirling flows does not allow one to physically correctly calculate the statistical characteristics of the flow using gradient transport models based on the introduction of an effective turbulent viscosity coefficient. In fact, the lack of effective viscosity models for swirling flows can be analytically demonstrated by modeling a fully developed rotating tube flow [7]. The measured velocity of the vortex in the pipe varies approximately as the square of the normalized radius (r^2), but the effective viscosity models give an exactly linear tangential velocity profile.

It was said above that the new two-fluid turbulence model is capable of describing anisotropic turbulence. Therefore, below we present numerical results based on this model for a turbulent fluid flow in a rotating tube, which is called the Poiseuille-Couette-Taylor flow. This flow is quite well studied experimentally in [8–10]. Therefore, numerical results are compared with experimental data.

Setting the task.

The physical formulation of the problem is shown in Figure 1. As can be seen from the figure, a laminar non-swirling flow enters the rotating pipe, and the flow at the exit is completely turbulent and swirling. Therefore, for a sufficiently long pipe is considered, i.e. length is much greater than its diameter. The flow under consideration is characterized by the Reynolds number (Re=20000), which is determined by the average flow rate and the radius of the pipe. Under rotation conditions, a rotation parameter is also introduced $N = \frac{\pi D^2 \Omega R}{4Q}$, where ρ is the fluid density, Q is the volume flow, μ is the dynamic viscosity, and R is the pipe radius.

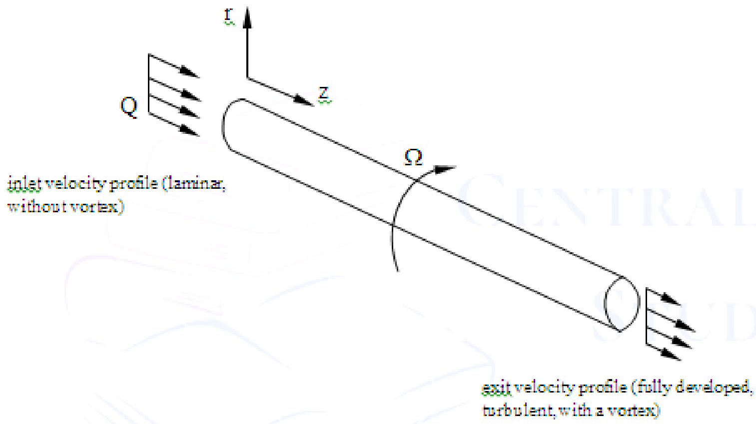

Fig. 1: Diagram of a swirling flow through a rotating pipe.

MATHEMATICAL SIMULATION OF THE PROBLEM

For numerical simulation of the turbulent flow of an incompressible fluid, the Reynolds equations were used:

$$\begin{cases} \dfrac{\partial \overline{U}_i}{\partial x_i} = 0, \\ \dfrac{\partial \overline{U}_i}{\partial t} + \overline{U}_j \dfrac{\partial \overline{U}_i}{\partial x_i} + \dfrac{1}{\rho}\dfrac{\partial \overline{p}}{\partial x_i} = \dfrac{\partial}{\partial x_i}\left[\nu \left(\dfrac{\partial \overline{U}_i}{\partial x_j} + \dfrac{\partial \overline{U}_j}{\partial x_i} \right) \right] + \dfrac{\partial (-\overline{u'_j u'_i})}{\partial x_i}. \end{cases} \qquad (1)$$

The system of Navier-Stokes equations averaged over Reynolds (1) is not closed. For closure in methods, non-linear turbulence approaches are used.

The Mixed SSG/LRR Second Moment Reynolds Stress Model is a non-linear RANS turbulence model that uses the omega equation for the length scale equation. Total second moment Reynolds stress models are very different from simpler linear or non-linear single equation models, as the latter use a constitutive relation giving Reynolds stresses τ_{ij} in terms of other tensors via some assumed relation (such as the Boussinesq conjecture). On the other hand, full second moment Reynolds stress models compute each of the

6 Reynolds stresses directly (the Reynolds stress tensor is symmetric, so there are 6 independent terms). Each Reynolds stress has its own transport equation. There is also a seventh transport equation for the scale variable. The complete Reynolds stress model SSG/LRR-omega (SSG/LRR-RSM-w2012) [11–13] and one length scale equation are:

$$\begin{cases} \dfrac{\partial R_{ij}}{\partial t}+\dfrac{\partial \overline{U}_k R_{ij}}{\partial x_k}=P_{ij}+\Pi_{ij}-\varepsilon_{ij}+D_{ij}, \\ \dfrac{\partial \omega}{\partial t}+\dfrac{\partial \overline{U}_k \omega}{\partial x_k}=\dfrac{a_\omega \omega}{k}\dfrac{P_{kk}}{2}-B_\omega \omega^2+\dfrac{\partial}{\partial x_k}\left(\left(\mu+\sigma_\omega \dfrac{k}{\omega}\right)\dfrac{\partial \omega}{\partial x_k}\right)+\sigma_d \dfrac{1}{\omega}\dfrac{\partial k}{\partial x_j}\dfrac{\partial \omega}{\partial x_j}. \end{cases} \tag{4}$$

Here $\rho R_{ij}=-\tau_{ij}=\rho\overline{u_i' u_j'}$, P_{ij} is the generation of Reynolds stresses, D_{ij} is diffusion, ε_{ij} is dissipation, and Π_{ij} is the pressure redistribution term.

The generation of Reynolds stresses is modeled using:

$$P_{ij}=-R_{ik}\frac{\partial \overline{U}_j}{\partial x_k}-R_{jk}\frac{\partial \overline{U}_i}{\partial x_k}.$$

The dissipation has the form:

$$\varepsilon_{ij}=\frac{2}{3}\varepsilon\delta_{ij}.$$

Where $\varepsilon=C_\mu k\omega$ and $k=R_{ii}/2$.

The pressure-strain correlation is modeled using the relationship:

$$\Pi_{ij}=-\left(C_1\varepsilon+\frac{1}{2}C_1^* P_{ij}\right)a_{ij}+C_2\varepsilon\left(a_{ik}a_{kj}-\frac{1}{3}a_{kl}a_{kl}\delta_{ij}\right)+\left(C_3-C_3^*\sqrt{a_{kl}a_{kl}}\right)kS_{ij}^*+$$
$$+C_4 k\left(a_{ik}S_{jk}+a_{jk}S_{ik}-\frac{2}{3}a_{kl}S_{kl}\delta_{ij}\right)+C_5 k\left(a_{ik}W_{jk}+a_{jk}W_{ik}\right),$$

where the anisotropy tensor of the Reynolds stresses is given by:

$$a_{ij}=\frac{R_{ij}}{k}-\frac{2}{3}\delta_{ij}.$$

The pressure-strain ratios are mixed (as described below) between Launder-Reece-Rodi (LRR) near the walls (no wall correction terms) and Speziale-Sarkar-Gatski (SSG) away from the walls.

Also:

$$S_{ij}=\frac{1}{2}\left(\frac{\partial \overline{U}_i}{\partial x_j}+\frac{\partial \overline{U}_j}{\partial x_i}\right),\ S_{ij}^*=S_{ij}-\frac{1}{3}S_{kk}\delta_{ij},\ W_{ij}=\frac{1}{2}\left(\frac{\partial \overline{U}_i}{\partial x_j}-\frac{\partial \overline{U}_j}{\partial x_i}\right).$$

Neglecting the pressure diffusion component, the diffusion term is modeled using the generalized gradient diffusion model:

$$D_{ij}=\frac{\partial}{\partial x_k}\left(\left(\mu\delta_{kl}+D\frac{R_{kl}}{C_\mu \omega}\right)\frac{\partial R_{ij}}{\partial x_l}\right).$$

The rest of the initial and boundary conditions are presented in [14–16].

A new two-fluid model of Malikov's turbulence is presented in [8], which has the form [17–19]

$$\begin{cases} \dfrac{\partial \overline{U}_i}{\partial t}+\overline{U}_j\dfrac{\partial \overline{U}_i}{\partial x_j}=-\dfrac{1}{\rho}\dfrac{\partial \overline{p}_i}{\partial x_i}+\dfrac{\partial}{\partial x_j}\left[\nu\left(\dfrac{\partial \overline{U}_i}{\partial x_j}+\dfrac{\partial \overline{U}_j}{\partial x_i}\right)-u_j u_i\right], \\ \dfrac{\partial u_i}{\partial t}+\overline{U}_j\dfrac{\partial u_i}{\partial x_j}=-u_j\dfrac{\partial \overline{U}_i}{\partial x_j}+\dfrac{\partial}{\partial x_j}\left[\nu_{ij}\left(\dfrac{\partial u_i}{\partial x_j}+\dfrac{\partial u_j}{\partial x_i}\right)\right]+\dfrac{F_{fi}}{\rho}+\dfrac{F_{\perp i}}{\rho}, \\ \nu_{ij}=3\nu+2\left|\dfrac{u_i u_j}{def\left(\overline{U}\right)}\right|,\ i\neq j,\ \nu_{ii}=3\nu+\dfrac{2}{div(\upsilon\nu)}\left|\dfrac{u_k u_k}{def\left(\overline{U}\right)}\right|\dfrac{\partial \rho u_k}{\partial x_k}, F_f=-\rho K_f u, \\ \dfrac{\partial \overline{U}p_i}{\partial t}+\overline{U}_j\dfrac{\partial \overline{U}p_i}{\partial x_j}=k_m\left(\overline{U}_i-\overline{U}p_i\right),\quad F_\perp=\rho C_s rot\overline{U}\times u,\quad \dfrac{\partial \overline{U}_j}{\partial x_j}=0. \end{cases} \tag{5}$$

Here u_i are the axial, radial and tangential relative velocities, respectively, C_s =1 the Seffman force coefficient, ν_{ij} is the kinematic molar viscosity, $def\overline{U}$ is the deformation of the average flow velocity. The last equation is the kinematics equations for the solid phase [20–23].

Research results. On fig. 2. Numerical results of the non-linear SSG/LRR-RSM model and two-fluid model for the longitudinal velocity are attached. The results of the nonlinear turbulence approaches can be said to qualitatively describe the longitudinal velocity, while the two-fluid model describes quantitatively.

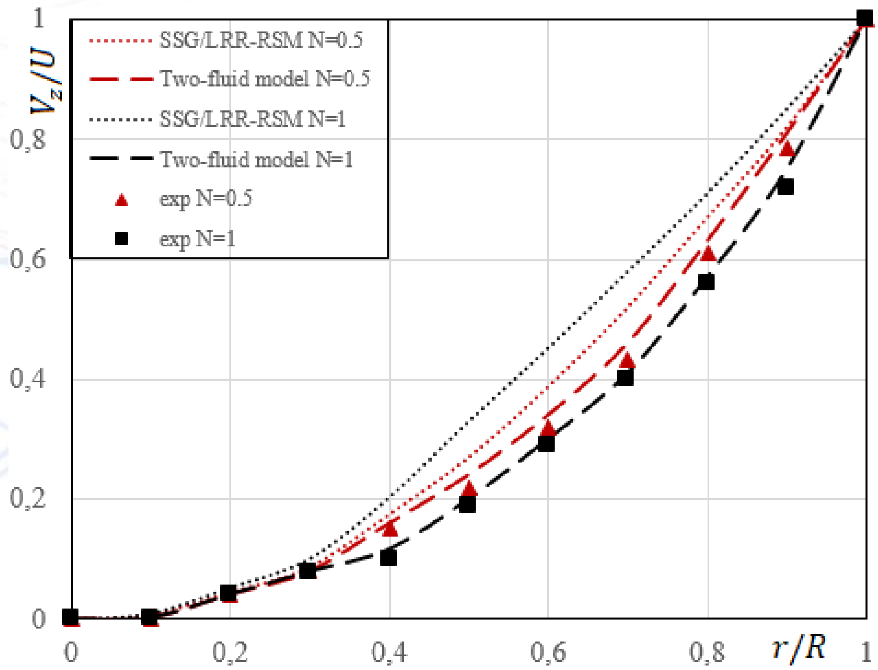

Figure 2. Axial velocity profile in a rotating tube

It can be seen from this figure that the nonlinear SSG/LRR-RSM model is not even qualitatively able to describe the tangential velocity, which confirms the above statements. As for the new model, we can observe a good agreement with the experimental data.

Conclusion.

1. The study showed that the Malikov`s two-fluid model has great potential for modeling turbulent swirling flows.
2. The model has demonstrated simplicity for numerical implementation and good stability.
3. A new model of profitability in terms of accumulation time. For example, compared to the SARC model, the new model allows you to integrate a time speed of 20 times more.

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