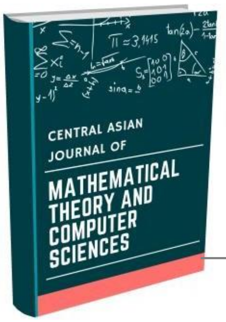

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Solving NP-Hard Problem With The Semidefinite Programming Field

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Abstract:

In this paper we have study the solution to the Maximum Independent Set optimization problem in semidefinite programming field. In fact, a new approach has been developed to replace the penalty method with the augmented Lagrangian method according to the value of the parameter. Also, a combined method that switches between the two methods was developed called the combined method. The proposed three approaches of the augmented Lagrangian problem, and the penalty problem were studied for the linear programming (LP) problems. As a result, only two approaches were justified and approved as valid methods to be used for solving the SDP relaxations. Finally, Julia language was applied to obtain the numerical results.

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1. INTRODUCTION

Nowadays the optimization principle has become well established as a guiding principle in the study of many complex decision or allocation problems. It gives a certain level of irreplaceable philosophical elegance, as well as a necessary level of operational simplicity. According to the concept of optimization, one addresses a complex choice problem that requires the selection of values for a number of related variables by focusing on a single goal aimed at evaluating performance and determining decision quality, and this goal is reduced or expanded, according to the constraints that may limit the values of the decision variables. Improvement applications take a wide range within the fields of scientific and practical life, as it allows setting goals if it is possible to isolate one appropriate aspect of the problem and describe it as a goal, whether it is profit or loss within the field of business, or measuring speed or distance in a material problem, or the expected return in The context of investments that require refining or blending industries are often risky. Moreover, it can provide a useful basis for analysis. When faced with a complex selection problem, it is unusual for all the intricacies of variable interactions, constraints, and goals to be effectively represented. As a result, as with every quantitative analysis technique, only a specific optimization formula should be considered as an approximation. As for the topic of our paper, we are interested in finding approximate solutions to the maximum independent set problem , as it is Np-hard problem and there is no precise one for it. Therefore, our idea is approach the solution through semidefinite programming using two methods:

(Penalty and Lagrangian), [1-6] .

2. The NP-Hard PROBLEM

The NP-hard problem is one of the hard problems that cannot be solved in non-deterministic polynomial time. In 1965 [7], John Hartmanis with Richard Stearns published a Turing Prize-winning paper on the computational complexity of algorithms. NP rises exponentially with the amount of the input in the basic technique, which was invented in 1947 by American mathematician George Dantzig. Leonid Khachyan, a Russian mathematician, devised the polynomial method in 1979. In August 2010 [8], Vinay Deolalikar, is believed to have solved the puzzle of P versus NP in a move that could change the way humans use computers. See Figure 1

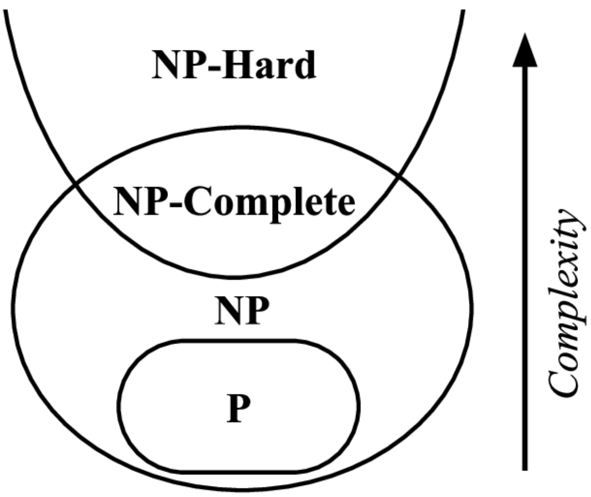

Figure 1. NP has P and it intersects the class of NP-hard problems. the

The intercept contains the NP-Complete problem

3. SEMIDEFINITE PROGRAMMING

During the 1990s, semidefinite programming (SDP) was one of the most active and intriguing fields of optimization study ([1], [2]). It has drawn professionals in convex programming, linear algebra, numerical optimization, combinatorial optimization, control theory, and statistics from a wide range of disciplines. The discovery of crucial applications in optimization and combinatorial theory, the invention of efficient internal point algorithms for solving SDP problems, and the depth and elegance of underlying optimization theory have all fueled this remarkable research effort. One of the important problems is the semi-deterministic programming problem, where the solution to semi-deterministic programming problems is still a difficult method and the difficulty of reaching the optimal solution [3]. In semi-deterministic programming, we minimize a linear function subject to the limitations imposed by the approximation. A semideterministic positive is a combination of identical matrices. Positive specified programs are convex optimization problems because such a constraint is nonlinear, smooth, and convex. Semi-deterministic programming unifies many standard problems (such as linear and quadratic programming) and has numerous technical applications. Despite being more general than linear programs, semi-deterministic programs are simple to solve.

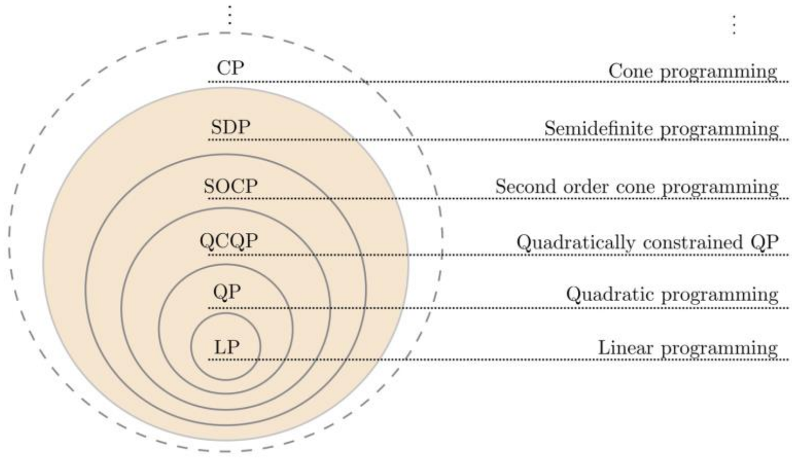

Figure 2. The relationship between (SDP) and other indicated fields.

4. MAXIMUM INDEPENDENT SET PROBLEM (MIS)

An independent set in a graph is a subset of vertices that are not contiguous to one another. The maximal independent set problem entails locating an independent set with the greatest cardinality in a graph [10,14,15]. In general, this problem is NP-hard. The maximal independent set (MIS) issue is an intriguing topic of combinatorial optimization with several applications in domains such as physics, computer science, and mathematics. This means that finding the exact solution is challenging. Several papers have been written about the MIS problem. Goemans-Williamson (1995) [9], initiated this line of research with their semidefinite programming relaxation-based approximation for the MIS issue. Poljak demonstrated that linear programming techniques are incapable of producing a superior approximation solution. Mathematically, For a graph G=(V,E), an independent set is S if and only if one of the following is true for v:

$$v \in S,\ N(v) \cap S \neq 0,\text{where } N(v) \text{ refer to neighbors of } v.$$

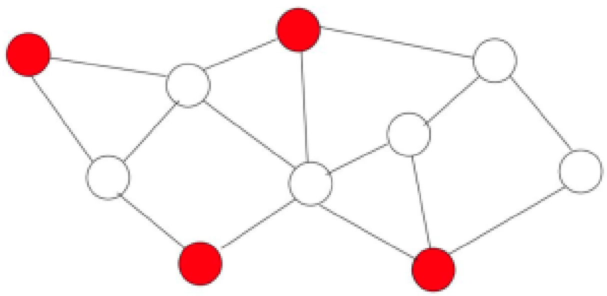

Figure 3. The maximum independent set (MIS) problem.

5. APPROXIMATE METHODS

Approximation methods are a hot topic in optimization. Suppose the convex function that minimizes $H: R^n \rightarrow R$ on a convex set D. The goal of approximation approaches is to replace H and D with approximation H^k and D^k. It should be noted that the use of the approximation approach will be possible only when reaching the goal is easier than the basic problem. That is every iteration k we tried to obtain $D^{k+1} = \arg\min_{d \in D^k} H^k(D)$ then in the following iteration, H^{k+1} and D^{k+1} are produced through approximation which depends on the new point D^{k+1}. Many great approximation methods are based on this idea, such as polyhedral approximation, the penalty method and the augmented Lagrangian method, and interior point methods. Our work focuses on the penalty and augmented Lagrangian methods. In general, In optimization problems, the presence of limitations complicates the algorithmic solution and narrows the range of possible solutions. As a result, it's only reasonable to try to remove constraints by approximating the relevant indicator function. For example, replace restrictions with penalty functions that impose a significant cost for breaking them [11,12]. The linear equality constrained problem is given by

$$\begin{aligned} \text{minimize} \quad & \langle c, d \rangle \\ \text{subject to} \quad & \langle a_i, d \rangle = b_i, i = 1, \dots, m, \\ & d \in D. \end{aligned}$$

The previous problem can be replaced by a penalized version

$$\begin{aligned} \text{minimize} \quad & \langle c, d \rangle + \alpha^k \sum_{i=1}^{m} P_q(\langle a_i, d \rangle - b_i) \\ \text{subject to} \quad & d \in D. \end{aligned}$$

The scalar α^k is a positive penalty parameter, and as α^k approaches zero, the penalized problem's solution d^k tends to lessen the constraint violation, providing an increasingly accurate approximation to the original issue. An key practical issue to note here is that k should be steadily decreased, with the optimal solution of each approximation problem used to begin iteration of the next approximating problem. The quadratic punishment function is one option for P_q, with the penalized problem (1.3) taking the form

$$\begin{aligned} \text{minimize} \quad & \langle c, d \rangle + \frac{1}{2\alpha^k} \| Ad - b \|^2 \\ \text{subject to} \quad & d \in D. \end{aligned}$$

On the assumption that $Ax = b$ represents the equation system $\langle a_i, d \rangle = b_i, i = 1, \dots, m$. The augmented Lagrangian method is a significant improvement over the penalty function approach, where we introduce a linear term into $P_q(y)$, incorporating a multiplier vector $y^k \in \mathbb{R}^n$. Then instead of problem (1.3), We are now finding a solution to the problem

$$\begin{aligned} \text{minimize} \quad & \langle c, d \rangle + \left(y^k\right)^T (Ad - b) + \frac{1}{2\alpha^k} \| Ad - b \|^2 \\ \text{subject to} \quad & d \in D. \end{aligned}$$

After obtaining the solution to the preceding problem x^k, the multiplier vector yk is updated using a formula that attempts to approximate an optimal dual solution [6]. such that

$$y^{k+1} = y^k + \frac{1}{\alpha^k}\left(Ad^k - b\right).$$

This is referred to as first-order Lagrangian augmentation methods , what it's called (the first order method of multipliers). For both inequality and equality requirements, penalty and augmented Lagrangian methods can be employed .

6. PROPOSED APPROACH TECHNIQUE

The semidefinite programming field is especially necessary because it has many fields, as shown in the figure 2. As a result, the idea arose to find a solution to the constant NP optimization problem through a semidefinite (SDP) bound as a bound to the maximum independent set (MIS), according to the mathematical relationship :

$$\text{MIS bond} \leq \text{SDP bond}$$

Of course, the solution will be approximate because, according to our predecessors previously, the problem does not have an exact solution, so here the function calls be adopted, through which a comparison will be made between the solution methods in the speed of each method reaching the specified bound, and this is what we will see through the numerical results in the next section.

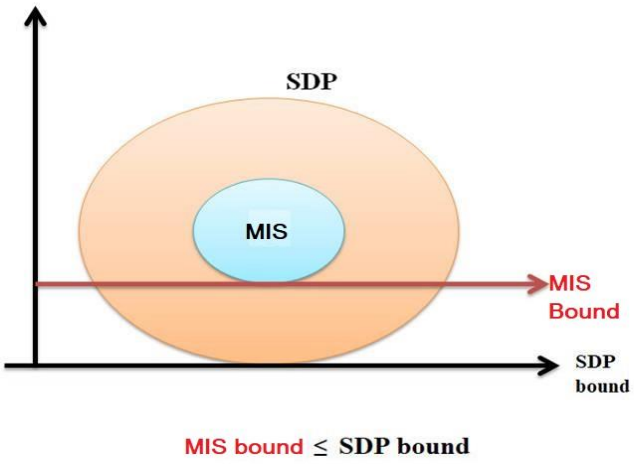

Figure 4. The idea of the solution by a simplified arrow diagram.

7. NUMERICAL RESULTES

Figure (5 and 6) of graph types (ising -80.0 and ising -100.0) demonstrates the convergence of a solution, as indicated in these data. Actually, the sample (ising -80.0) was the center of the test, and the augmented Lagrangian technique definitely outperforms the other two methods in terms of convergence (penalty and combined). As shown in table (1) the exact solution of the maximum independent set (MIS) in model (ising -80.0) looks to be 127, whereas the semidefinite bound is 129. The augmented Lagrangian method algorithm reached this bound in 380 function calls, whereas the combined technique took 451 function calls and the penalty method took 740 function calls, that is the penalty method required more time to reach the same constraint. On the same vein we see in the (ising -100.1) model, the combined method was the fastest to reach the maximum, according to the data of table (2).Since hitting the bound with minimal function calls saves time, the augmented Lagrangian and combined methods are the best for hitting the target. Finally, these graphs (ising -80.0 and ising -100.0) were chosen from the Biq Mac library [14,15].

Our results					
Graphs	The goal bound	Optimal value	Penalty f.calls	Aug f.calls	combined
ising80.0	129.04253	127	740	***380***	451
ising80.1	130.29090	126	534	**349**	*352*
ising80.2	126.60467	125	632	***355***	371
ising80.3	115.33249	111	528	394	***275***
ising80.4	131.42055	128	713	***441***	277
ising80.5	129.82046	128	552	405	***268***
ising80.6	125.54201	122	521	317	***264***
ising80.7	112.00673	112	923	***780***	870
ising80.8	121.17854	120	686	470	***379***
ising80.9	127.81774	127	1238	686	***640***

Table (1): The test types of graphs (pm1d 80 which has n=80) .

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ising 100.9	127.81774	127	1238	840	***686***

Table (2): The test types of graphs (ising 100 which has n=100) .

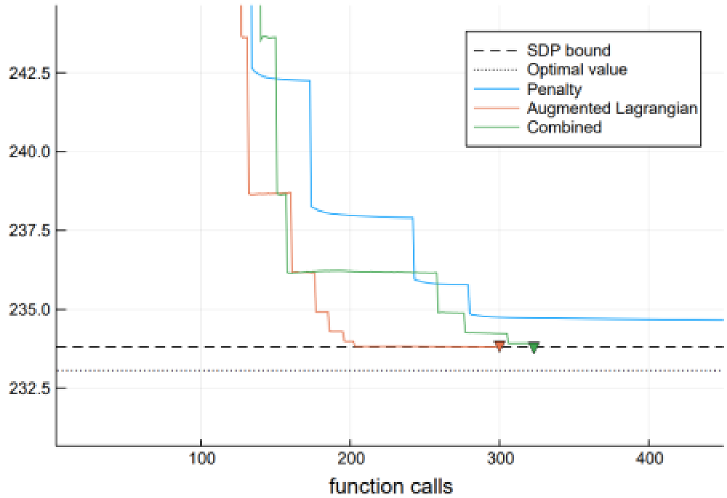

Figure 5. Bounds and function calls for graph ising -80.0.

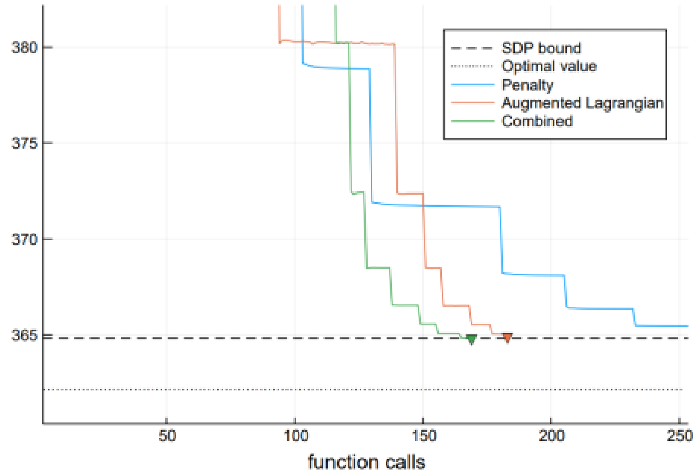

Figure 6. Bounds and function calls for graph 100.0.

8. CONCLUSION

The proposed approach was tested with a range of graphs from the Biq Mac library. These graphs have a wide range of properties, including a large number of nodes and edges. In terms of fulfilling the goal bound, the results showed that the combined and augmented lagrangian approach were preferred to the Penalty method. We also put to the test the combination technique, which switches between the penalty and inequality Augmented lagrangian methods. Finally, according to the data, the combined approach outperformed the two procedures separately.

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