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ON TEACHING STUDENTS NEWTON'S METHOD OF SOLVING ONE-DIMENSIONAL OPTIMIZATION PROBLEMS

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Abstract

We know that optimization is the selection of the best available options to achieve the highest efficiency in a process. One-dimensional optimization, that is, one-variable function optimization, is one of the simplest cases of optimization problems. Even so, such issues occupy an important place in the theory of optimization. The reason is that one-dimensional optimization problems are often encountered in engineering practice and are also widely used in complex multi-dimensional optimization problems.

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To solve the optimization problems, the steps are performed in the following sequence. let's say *f(x)* let the function be twice differentiable. As we know, the minimum condition of such a function:

$$f'(x^*) = 0$$

and this is a necessary condition. x^* - and for the point to be the minimum point:

$$f''(x^*) > 0$$

a condition is a sufficient condition.

So, $f'(x^*) = 0$ we solve the equation numerically. We give the initial approximation of xk, and we approximate the solution ourselves. We expand the function at this point into a Taylor series. In this case, we limit ourselves to the second-order term, that is, we build a quadratic model of the function.

$$f(x) \approx \bar{f}(x) = f(x_k) + f'(x_k)(x - x_k)$$

If $f''(x_k) \neq 0$ if so, then *f(x)* has a single stationary point. To find this point $\hat{f}'(x)$ – we make the derivative equal to zero:

$$f'(x) = 0; \hat{f}'(x) = \left(f(x_k) + f'(x_k)(x - x_k) + \frac{1}{2!}(f''(x_k)(x - x_k)^2)\right)' = 0$$

$$=> 0 + f'(x_k) + \frac{1}{2} * 2 * f''(x_k)(x - x_k) = 0 =>$$

$x = x_k - \frac{f'(x_k)}{f''(x_k)}$ find the solution to the minimum of x k+1 - we accept as approximation, as a result we have the following iterative formula:

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)} \qquad (*)$$

This formula is the same used to solve the equation *f(x)*=0

$$x_{k+1} = x_k - \frac{f\ (x_k)}{f'(x_k)}$$

It should be noted that it is similar to the formula of Newton's method, but differs from it.

1) $f'(x) = 0$ the equation can determine not only the minimum, but also the maximum point.

2) $\hat{f}(x)$ the model function may differ greatly from the function *f(x)* being optimized

That is why at every step to check that we are going to the minimum $f(x_{k+1})<f(x_k)$ we need to check the condition. If this condition is met, then we will proceed to the next step. If $f(x_{k+1})<f(x_k)$ and $f'(x)(x - x_k) < 0$ is, then *f(x)* must decrease from x_k in the direction of x_{k+1} at the beginning of the function. Therefore, the next optimal point can be found by dividing the step in the opposite direction, for example:

$$x'_{k+1} = \frac{x_{k+1} + x_k}{2}$$

as can be seen from the formula (∗), $f'(x_k)(x - x_k)$ the expression is negative if and only if $f''(x_k)>0$. This means that if the local model used to derive the Newton step has a minimum (not a maximum), then the step is guaranteed to have an optimal direction.

If on the other hand, if $f''(x) < 0$and $f'(x_k)(x - x_k) >0$, then *f(x)* increases initially when going from x_k to x_{k+1}, so the step should be done in the opposite direction.

As a criterion for stopping iteration (convergence) in optimization.

$$\left|\frac{f'(x_{k+1})}{f(x_k)}\right| < \varepsilon$$

the condition can be accepted, ε - pre-given precision. This method is called Newton or Newton-Raphson method. In some cases *f(x)* it is difficult to get derivatives of the function, in such cases Newton's method can be modified. For this, we choose initial approximation x_k and step h. x_k-h, x_k, x_k+h let's look at the points. In that case $f'(x_k)$ va $f''(x_k)$ derivatives can be replaced by approximate formulas as follows.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \quad \text{or} \quad f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x-h)}{2h}$$

lim dropping the symbols, we get the following approximate formulas

$$f'(x_k) \approx \frac{f(x_k+h) - f(x_k-h)}{2h};$$

$$f''(x_k) \approx \frac{f'(x_k+h) - f'(x_k-h)}{2h} = \frac{\frac{f(x_k+h)-f(x_k)}{h} - \frac{f(x_k)-f(x_k-h)}{h}}{2h} =>$$

$$f''(x_k) \approx \frac{f(x_k+h) - 2f(x_k) + f(x_k-h)}{2h^2}.$$

In that case, if we put (*) in the above formula, we will get an iterative formula as a result:

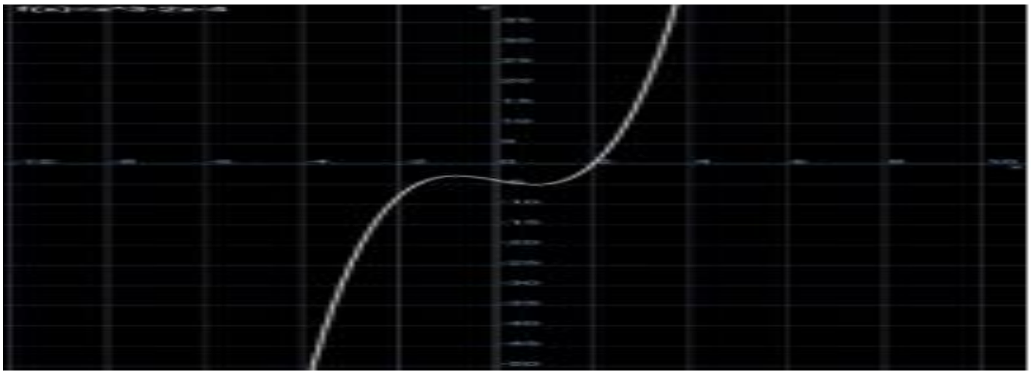
$$x_{k+1} = x_k - \frac{f(x_k+h) - f(x_k-h)}{f(x_k+h) - 2f(x_k) + f(x_k-h)} * h$$

This formula is called Quasi Newton or Newton's modified method.

Let's look at the following examples:

1) $f(x) = x^3 - 2x - 4$

x_1 we look at the graph of the function to pick a point. As can be seen from the graph, the minimum point is close to the point x=1. That is



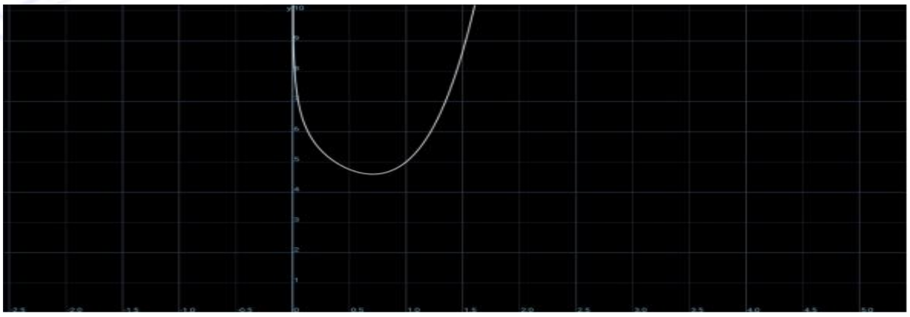
$x_1 = 1$ we choose that. Using the above formulas successively, we find the solution with the given accuracy. We use the Excel program to facilitate calculations:

x_k	Values	$f'(x_k)$	$f''(x_k)$	$f(x_k)$	?	$f(x_{k+1})$	$\left\|\frac{f'(x_{k+1})}{f(x_k)}\right\| < 0,001$
x_1	1	1	6	-5	>	-5,087962963	0,016666667
x_2	0,833333333	0,083333333	5	-5,087962963	>	-5,088662037	0,000163785
x_3	0,816666667	0,000833333	4,9	-5,088662037	>	-5,088662108	1,70515E-08

Since we selected a point $x_1$using the graph, the solution was found in the second step. x_1 the number of steps may increase when we choose a point arbitrarily.

2) Let's look at the graph of the following $f(x) = x^4 - \ln(x) + 4$. We find the first and second derivatives of the function:

$$f'(x) = 4x^3 - \frac{1}{x}; \quad f''(x) = 12x^2 + \frac{1}{x^2}$$



Using the graph, we select the point x_1=0,7. x_1 since we chose a point very close to the exact solution, the solution was found in two steps with the given accuracy.

x_k	Values	$f'(x_k)$	$f''(x_k)$	$f(x_k)$	?	$f(x_{k+1})$	$\left\|\frac{f'(x_{k+1})}{f(x_k)}\right\| < 0,001$
x_1	0,7	-0,056571429	7,920816327	4,596774944	>	4,596573595	6,15052E-05
x_2	0,707142121	0,000282726	8,000399854	4,596573595	>	4,59657359	1,53706E-09

Newton's method used to solve optimization problems has the following disadvantages:

1) The method requires a sufficiently good initial approximation, otherwise the number of steps will increase, that is, the selected point may have a large difference in the minimum of the function *f(x)* being optimized
2) It requires the existence and analyticity of first and second order derivatives.
3) In Newton's method, there is no obstacle that prevents the iterative method from going towards maximum or turning points instead of the required minimum point. Also, the possibility that the step x_{k+1} -x_k - is too large can be shown as a disadvantage.

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