

SOME PROBLEMS OF MECHANICS AND PHYSICS LEADING TO THE SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS

Baymuratova Klara Amangeldieva

Karakalpak State University named after Berdakh

Abstract

This article discusses some problems of mechanics and physics that lead to the solution of ordinary differential equations. The calculation of currents and voltages in electrical circuits using differential equations is a classic method for calculating circuits in electrical engineering. Mathematical analysis fully realizes itself in applied problems through differential equations.

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Differential equations, which are one of the main sections of higher mathematics, are unique in content and universal in their use, means of knowing and understanding this or that phenomenon, their effective use and increasing the reliability of the results obtained. Problems leading to differential equations began to be studied by scientists at the beginning of the 17th century. For example, R. Descartes studied a plane curve based on the properties of its tangents. All of them arrived at the simplest first-order differential equations with separable variables. Initially, the theory of differential equations developed within mathematical analysis. Only gradually, as the features of its problems were clarified and its central concepts established, did it emerge as a special mathematical science. Mathematical analysis fully realizes itself in applied problems through differential equations.[4] This is natural, since the derivative of the function determines such practical concepts as the slope of the tangent to the curve, the instantaneous speed of movement, etc. The second derivative makes it possible to determine the curvature of the curve, acceleration, etc. Differential equations relate the desired function, its derivatives of different orders and independent variables. Differential equations are undoubtedly the most important mathematical apparatus in modeling processes occurring in nature and society.

Let's consider some problems of mechanics and physics leading to the solution of ordinary differential equations, which it is advisable to consider with students.

1. The problem of the free fall of a body. Let a body of mass m be dropped from a certain height H [1]

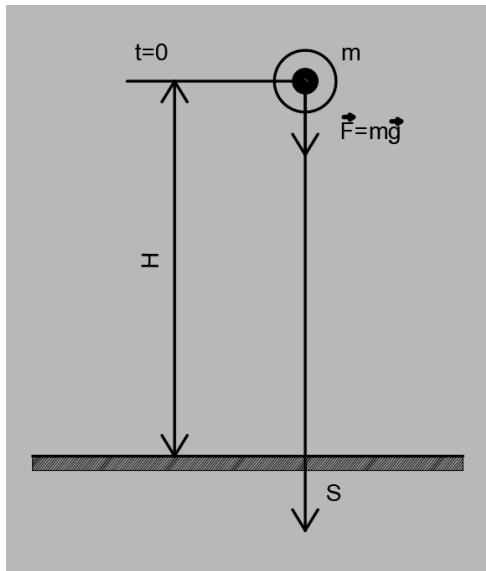


Fig.1

It is required to establish how long the body will reach the earth's surface (neglect air resistance).

It is clear from the condition that the body moves under the action of gravity $\vec{F} = m\vec{g}$. Let us direct the axis s of the reference of the body movement vertically downwards so that its beginning coincides with the initial position of the body. According to Newton's second law, we have

$$m \frac{d^2s}{dt^2} = mg \quad (1)$$

where m is the mass of the body, $\frac{d^2s}{dt^2}$ is the acceleration of the moving body (the second derivative of the displacement with respect to time) g - is the acceleration of free fall. Equation (1) is a second order differential equation. Reducing by m , we get $s'' = g$. Solving this equation, we find

$$s' = gt + C_1, s = \frac{gt^2}{2} + C_1t + C_2 \quad (2)$$

If at the initial moment of time $t = 0$ the speed and displacement respectively were equal to v_0 and s_0 , then from equations (2) we obtain $v_0 = C_1$, $s_0 = C_2$ then the law of motion of the body will take the form

$$s = \frac{gt^2}{2} + v_0 t + s_0$$

If in equality (3) $v_0 \neq 0, s_0 = 0$, then we obtain the relation known from mechanics

$$s = \frac{gt^2}{2} + v_0 t.$$

If $v_0 = 0$ and $s_0 = 0$, then $s = \frac{gt^2}{2}$. Substituting now in equality (3) the values

$s = H, v_0 = 0, s_0 = 0$, we obtain a formula for determining the free fall time of a body

$$t = \sqrt{\frac{2H}{g}}.$$

2. The problem of the transient process in an electrical circuit. In electrical circuit containing active resistance R , inductance L and electromotive force E , at the time $t = 0$ the switch P closes. Find the law by which the current i changes in this circuit.[5]

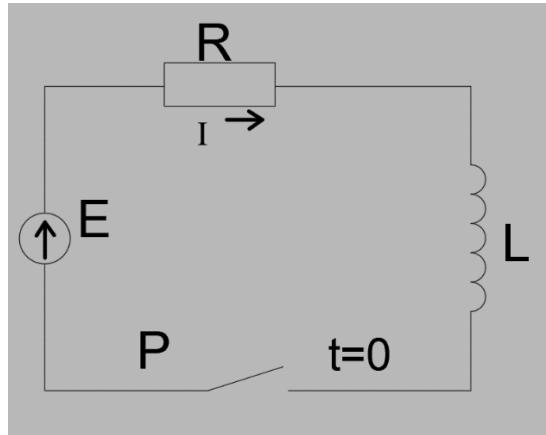


Fig.2

According to Ohm's law for a section of the circuit, the voltage drop across the active resistance will be R . When the circuit is closed in the coil L , an electromotive force of self-induction arises, directed opposite to the current i and proportional to the derivative $\frac{di}{dt}$, and the proportionality coefficient is equal to L .

According to the second Kirchhoff law for RL-chain for $t > 0$ we have $Ri = E - L\frac{di}{dt}$, whence

$$L\frac{di}{dt} + Ri = E \quad (4)$$

Equation (4) is a first-order linear differential equation with constant coefficients. By direct substitution, one can check that the general solution of equation (4) is the function

$$i = \frac{E}{R} + \frac{1}{CR}e^{-(R/L)t}, \quad (5)$$

Where C - is an arbitrary constant. Considering that at $t = 0$ there is no electric current in the circuit ($i=0$), we have

$\frac{E}{R} + \frac{1}{CR} = 0$ whence $C = -\frac{1}{E}$. Substituting the value of C - into equation (5), we obtain the law of current change in the RL- circuit

$$i = \frac{E}{R}(1 - e^{-(R/L)t}), \quad (6)$$

In formula (6), the term $e^{-(R/L)t}$ decreases with increasing t . Thus, the steady value of the current after a sufficiently long period of time has elapsed from the moment the RL-circuit is closed is determined by the value of E/R

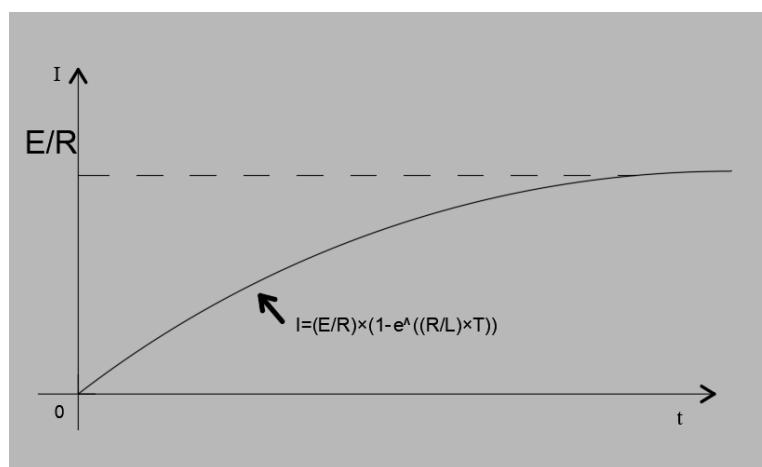


Fig.3

Note that the calculation of currents and voltages in electrical circuits using differential equations is a classic method for calculating circuits in electrical engineering.

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