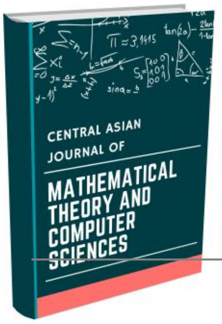

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THE EXACT SOLUTION OF PARTIAL DIFFERENTIAL EQUATIONS

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Abstract

In this paper, we obtain the exact solutions of fractional differential equation with Atangana-Baleanu fractional derivative (ABFD) by using fractional Adomian decomposition method (FADM). The exact solutions of PDEs with fractional order are successfully obtained using the proposed method.

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1. Introduction

Fractional differential equations (FDEs) have gained a lot of attention of researchers due to their ability to enhance real-world issues, used in various fields of engineering and physics. Numerous physical marvels in signal processing, chemical physics, electrochemistry of corrosion, probability and statistics, acoustics and electromagnetic are precisely modeled by DEs of fractional order [1]. Nonlinear partial differential equations (NPDEs) can be considered the generalization of the differential equations of integer order [2]. In the modern age it is impossible to imagine modeling of many real world problems without using fractional partial differential equations (FPDEs). Indeed, fractional calculus can be called this century's calculus [3] because of the diversity of applications in different areas of science and technology. Many numerical and analytical techniques have been suggested for the solutions of linear and nonlinear FPDEs. Some emerging analytical approximate approaches for FDEs are Adomian decomposition method, homotopy analysis method, variational iteration method, homotopy analysis transform method, reduce differential transform method, differential transform method, Sumudu variational iteration method, Laplace homotopy perturbation method (LHPM), and Laplace variational iteration method and Sumudu homotopy perturbation method [4-93].

2. Preliminaries

Definition 2.1 The Atangana-Baleanu fractional derivative (ABFD) of order α defined as follows [36]:

$$ {}_{a}^{AB}D_{t}^{\alpha}u(t) = \frac{M(\alpha)}{1-\alpha}\int_{a}^{t} E_{\alpha}\left(\frac{-\alpha(t-x)^{\alpha}}{\alpha-1}\right) u'(x)dx \quad (2.1) $$

where $0 < \alpha < 1$ and $M(\alpha)$ is a normalization function, such that $M(0) = M(1) = 1$.

Definition 2.1. The Atangana-Baleanu fractional integral (ABFI) of order α defined as follows [36]:

$$ {}_{a}^{AB}I_{t}^{\alpha}u(t) = \frac{1-\alpha}{M(\alpha)}u(t) + \frac{\alpha}{M(\alpha)}\frac{1}{\Gamma(\alpha)}\int_{a}^{t}(t-x)^{\alpha-1}u(x)\,dx \quad (5) $$

3. Analysis of FADM.

Let us consider the following fractional partial differential equation:

$$ {}^{AB}_{a}D^{\alpha}_{t}\, u(x,t) + R\, u(x,t) + N\, u(x,t) = g(x,t), 0 < \alpha < 1, t > 0, (3.1)$$

with the initial condition

$$u(x,0) = f(x), (3.2)$$

where ${}^{AB}_{a}D^{\alpha}_{t}\, u(x,t)$ Atangana-Baleanu fractional operator.

The method is based on applying the operator ${}^{AB}_{a}I^{\alpha}_{t}$, the inverse of the operator ${}^{AB}_{a}D^{\alpha}_{t}$, on both sides of Eq. (3.1) to obtain

$$ {}^{AB}_{a}I^{\alpha}_{t}\,\{ {}^{AB}_{a}D^{\alpha}_{t}\, u(x,t)\} = {}^{AB}_{a}I^{\alpha}_{t}[g(x,t)] - {}^{AB}_{a}I^{\alpha}_{t}\,[R\, u(x,t) + N\, u(x,t)],$$

or equivalent

$$u(x,t) = u(x,0) + {}^{AB}_{a}I^{\alpha}_{t}\,[g(x,t)] - {}^{AB}_{a}I^{\alpha}_{t}\,[R\, u(x,t) + N\, u(x,t)]. (3.4)$$

The infinite series shown here reflects the ADM solution of $u(x,t)$ as

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t),\ (3.5)$$

The problem's nonlinear term may be written as an Adomian polynomial as follows:

$$N\, u(x,t) = \sum_{n=0}^{\infty} A_n, (3.6)$$

where

$$A_n = \frac{1}{n!}\left[\frac{\partial^n}{\partial \omega^n} N\left(\sum_{i=0}^{n} \omega_i u^i\right)\right]_{\omega=0}.$$

By adding Eq. (3.5) and Eq. (3.6) in Eq. (3.4), we get

$$\sum_{n=0}^{\infty} u_n(x,t) = f(x) + {}^{AB}_{a}I^{\alpha}_{t}\,[g(x,t)] - {}^{AB}_{a}I^{\alpha}_{t}\left[R\left(\sum_{n=0}^{\infty} u_n\right) + \sum_{n=0}^{\infty} A_n\right]. (3.7)$$

Following the ADM, we introduce the recursive relations as

$$u_0(x,t) = f(x) + {}^{AB}_{a}I^{\alpha}_{t}\,[g(x,t)],$$

$$u_{n+1}(x,t) = -{}^{AB}_{a}I^{\alpha}_{t}\,[R\,(u_n) + A_n], n \geq 0. (3.8)$$

As a result, the series solution is given by

$$u(x,t) = u_0 + u_1 + u_2 + \cdots$$

4. Applications

Example: Consider the following fractional differential equation

${}^{AB}D^{\alpha}_{t}u = u_{xx} - uu_{x}$ (4.1)

With initial condition

$u(x,0) = 2\tan x, 0 < \alpha \leq 1$

Taking ${}^{AB}I^{\alpha}_{t}u$ to both sides we get

$u(x,t) = 2\tan x + {}^{AB}I^{\alpha}_{t}\{u_{xx} - uu_{x}\}$

Using FADM for linear and nonlinear terms

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t)$$

Where A_n polynomilas for the nonlinear terms

$u_0(x,t) = 2\tan x$

$u_{n+1}(x,t) = {}^{AB}I_t^{\alpha}\{(u_n)_{xx} - A_n\}$

$A_0 = u_0 u_{0x} = 2\tan x\,[2\sec^2 x\,]$

$= 4\tan x\sec^2 x$

$u_1 = {}^{AB}I_t^{\alpha}\{(u_0)_{xx} - A_0\} = {}^{AB}I_t^{\alpha}\{4\tan x\sec^2 x - 4\tan x\sec^2 x\}$

$= 0$

$\vdots$

Then, we have

$$u\,(x,t) = \sum_{n=0}^{\infty} u_n(x,t) = 2\,tan\,x.$$

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