



## OPTIMIZATION APPROACH OF SIMPLE SPECIAL MATRICES INVERSE ESTIMATION

**Viean Abdulmuhsin Zainalaabdeen AL-Salihi**

Control and Systems Engineering Department, University of Technology, Iraq

[Viean.A.Zainalaabdeen@uotechnology.edu.iq](mailto:Viean.A.Zainalaabdeen@uotechnology.edu.iq)

### Abstract

*In this research the mathematical approach has been adopted to find out a matrix inverse depending on many trials of special matrix pattern. Special solution has been suggested in order to guess an inverse for studied matrices. Thus, this pattern may leads to new other approaches that reflected positively on the concepts of matrices.*

### ARTICLE INFO

#### Article history:

Received 3 Jun 2023

Revised form 5 Jul 2023

Accepted 4 Aug 2023

**Keywords:** Inverse, Optimization, Special matrix.

© 2023 Hosting by Central Asian Studies. All rights reserved.

### Introduction:

The study of matrices occupies a unique place within mathematics. It is still an area of active research, and utilized by every mathematician as well as many scientists working in various specialties. Further, its versatility is evident e.g. the generalized inverse is involved in least-squares approximation [1]. Several researches have been carried out for more understanding regarding matrices and as summarized hereunder:

Tieplova investigated the behaviour of the singular values of two kinds of structured large random matrices, and to use the corresponding results to address an important statistical problem [2,3]. Theint and Soe focused on the applications of matrices in human's life and multiplication of two matrices besides the determinant of a matrix and the corresponding eigen-values and eigen-vectors. Hence, the inverse of a given matrix had obtained by Gauss's Elimination and Gauss-Jordan [4]. Meanwhile, Lal and Pati studied several topics concerning matrices for more clarified concepts e.g. system of linear equation, linear transformations, and advanced topics on diagonalizability and triangularization [5]. Also, Hiai F. and Petz D concentrated on matrix analysis and applications e.g. matrix monotone functions and convexity, matrix means and inequalities, majorization and singular values [6,7]

Optimization is a method to find the best possible solution among many potential solutions and satisfying the chosen criteria [8,9]. In the other words, an optimization model provides feasible solutions that are easier than previous solutions. However, matrix inverse shall be found out promptly as much as possible and this is the key point that must be kept in mind while studying the optimization concepts and methods. Aforementioned studies have been conducted due it matrices importance and applications where this topic may be utilized in write, encodes, decode and mathematics puzzles, games, financial information like credit card number, bank account and all related encode, decode....etc. The aim of this study is to focus on more facts regarding special matrices that may lead for more investigation and optimization with regards to matrices subjects and its inverses.

**Special matrices (case study):**

The following special matrices has been studied in order to find several facts regarding its inverses

**Case1:**

$$X = \begin{bmatrix} x & 2x & 2x \\ 3x & x & 2x \\ 3x & 3x & x \end{bmatrix}$$

The feature of the matrix(case1) is the diagonal have the same values and the upper triangle above the diagonal have values equaled to two times diagonal value on the other hand the lower triangle have values three times diagonal values.

**Case2:**

$$X = \begin{bmatrix} x & 0 & 0 \\ 3x & x & 0 \\ 3x & 3x & x \end{bmatrix}$$

The feature of the matrix(case2) is the diagonal have the same values and the upper triangle above the diagonal have values equaled to zero on the other hand the lower triangle have values three times diagonal values.

**Case3:**

$$X = \begin{bmatrix} x & 2x & 2x \\ 0 & x & 2x \\ 0 & 0 & x \end{bmatrix}$$

The feature of the matrix(case1) is the diagonal have the same values and the upper triangle above the diagonal have values equaled to two times diagonal value on the other hand the lower triangle have zero values.

$$Y = X^{-1}$$

**Results and discussion:**

Matrix inverse for the special matrix has been found out with unique pattern and as illustrated hereunder:

**Case1:**

$$X = \begin{bmatrix} x & 2x & 2x \\ 3x & x & 2x \\ 3x & 3x & x \end{bmatrix}$$

$$Y = X^{-1}$$

$$Y = \begin{bmatrix} -y & 0.8y & 0.4y \\ 0.6y & -y & 0.8y \\ 1.2y & 0.6y & -y \end{bmatrix}$$

Where:

$$y = \frac{0.3846}{x}$$

A little consideration will show that above fact has been resulted depending on many trials for x values from (1), (2),.....to (7) and as indicated hereunder:

$$X = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 2 \\ 3 & 3 & 1 \end{bmatrix}$$

$$Y=X^{-1}$$

$$Y=\begin{bmatrix} -0.3846 & 0.3077 & 0.1538 \\ 0.2308 & -0.3846 & 0.3077 \\ 0.4615 & 0.2308 & -0.3846 \end{bmatrix}$$

.  
.  
.  
.  
.  
.

$$X=\begin{bmatrix} 7 & 14 & 14 \\ 21 & 7 & 14 \\ 21 & 21 & 7 \end{bmatrix}$$

$$Y=X^{-1}$$

$$Y=\begin{bmatrix} -0.0549 & 0.044 & 0.022 \\ 0.033 & -0.0549 & 0.044 \\ 0.0659 & 0.033 & -0.0549 \end{bmatrix}$$

Similarly, case1 and case2 have also been investigated by many trials and the following results have been found respectively:

#### Case2:

$$X=\begin{bmatrix} x & 0 & 0 \\ 3x & x & 0 \\ 3x & 3x & x \end{bmatrix}$$

$$Y=X^{-1}$$

$$Y=\begin{bmatrix} y & 0 & 0 \\ -3y & y & 0 \\ 6y & -3y & y \end{bmatrix}$$

Where:  $y = \frac{1}{x}$

#### Case3:

$$X=\begin{bmatrix} x & 2x & 2x \\ 0 & x & 2x \\ 0 & 0 & x \end{bmatrix}$$

$$Y=X^{-1}$$

$$Y=\begin{bmatrix} y & -2y & 2y \\ 0 & y & -2y \\ 0 & 0 & y \end{bmatrix}$$

Where:

$$y = \frac{1}{x}$$

It worthy to mention that many matrices (by the same form of aforementioned X matrices) have been solved and inverses for these matrices were tested and complied with aforementioned Y matrix. Thus, the solution has been verified.

### Conclusions:

1. The suggested solution regarding inverses of matrices that have the same form of X matrices yield excellent results with very good time for solution that may leads to more facts and conclusions within similar topics.
2. The values of (y) are equaled in (case1) and (case2).

### References:

1. Axler S. and et al, "Matrices: Theory and Applications" USA 2002.
2. AL-Salihi VA "New Formulas of Special Singular Matrices" Baghdad Science Journal. 2009;6 (2):405-409.
3. Tieplova D. "Application of large random matrices to multivariate time series analysis", 2020.
4. AL-Salihi VA "minimizing total tardiness and quadratic maximum Earliness on a single machine scheduling problem using l" Journal of college education. 2014: 3(3), 233-251.
5. Theint D. and Soe S., " Application of Matrices in Human's Life", International Journal of Science and Engineering Applications . 2019; 8(10),438-443.
6. AL-Salihi VA " Solving the Weighted Sum of Squares Completion Time on Single Machine Problem" Journal of College of Education. 2013.103-124.
7. Lal A. K. and Pati S., "Linear Algebra through Matrices", 10 July 2018.
8. Hiai F. and Petz D., "Introduction to Matrix Analysis and Applications" April 2013.
9. Salam J.H., W.H. Wan Badaruzzamana " Cost Optimisation of Water Tanks Designed According to The Aci and Euro Codes " 2015.