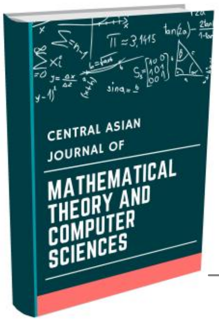

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Stress-strain state of thick shells of variable thickness

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ABSTRACT

This paper makes analyses of the stress-strain state of thick shells of variable thickness. On this case, research has been undergone from theoritical and methodological points as the whole. Outcomes and shortcomings were highlighted as conclusion of the research paper.

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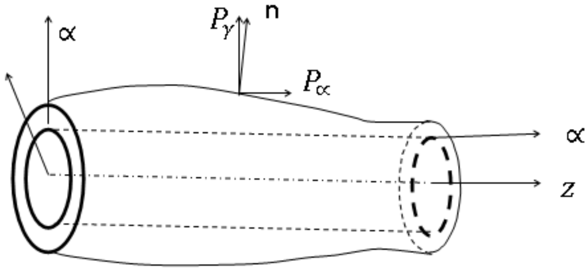
1.INTRODUCTION

Consider a thick-walled cylindrical shell, the relative thickness of which is an arbitrary function of the axial coordinate α. Let the shell be under the action of axisymmetric surface forces applied to the outer side surface(figure 1). We assume that the loads have both a vertical Pγ and a horizontal Pα component.



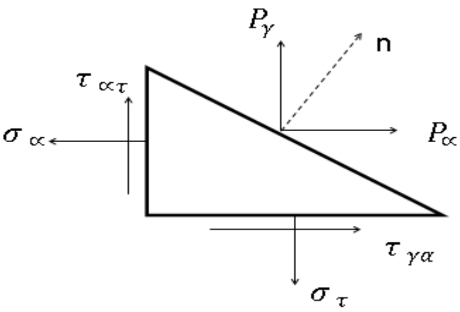


Figure 1

In addition, the inner surface of the shell is a circular cylindrical surface. The ends of the shell are free from loads (figure 2). As is known, stresses on an inclined platform are determined by the formulas:

$$P\alpha = 6\alpha Cos(n,\alpha)+\tau\alpha\gamma Cos(n,\gamma) \quad (1)$$

$$P\gamma = \tau\alpha\gamma Cos(n,\gamma)+ 6\gamma Cos(n,\gamma)$$

Guide cosines in expressions (1) are determined by the following relations:

$$Cos(n,\alpha)= \frac{dh}{d\alpha}\frac{1}{\sqrt{1+\left(\frac{dh}{d\alpha}\right)^2}} \quad ,$$

$$\mathrm{Cos}(n,\gamma) = -\frac{1}{\sqrt{1+\left(\frac{dh}{d\alpha}\right)^2}} \qquad (2)$$

PA =6αcos(n, α)+ταγCos(n,γ) (1) Pγ

= ταγCos(n,γ)+ 6γcos (n,γ) the

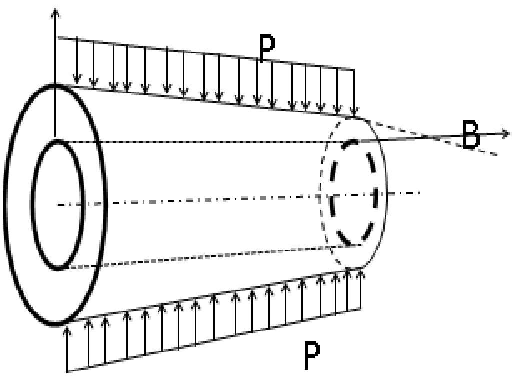


Figure 2

Then the equations (1) relating to the points of the boundary surface h = h (α) and being static conditions of equilibrium for this surface will take the form:

$$\left.\begin{aligned} P_{\alpha} &= \frac{1}{\sqrt{1+\left(\frac{dh}{d\alpha}\right)^2}}\left(\frac{dh}{d\alpha}\sigma_{\alpha} - \tau_{\alpha\gamma}\right) \\ P_{\gamma} &= \frac{1}{\sqrt{1+\left(\frac{dh}{d\alpha}\right)^2}}\left(\frac{dh}{d\alpha}\tau_{\alpha\gamma} - \sigma_{\gamma}\right) \end{aligned}\right\} \qquad (3)$$

Voltage 6γ, ταγ and 6α included in the formula(1) for internal points of the considered body are determined by the known ratio by the method of initial functions(MNF):

$$\left.\begin{aligned} \sigma_{\gamma} &= L_{31}U^0 + L_{32}W^0 + L_{33}\delta_{\gamma}^0 + L_{34}\tau_{\alpha\gamma}^0 \\ \tau_{\alpha\gamma} &= L_{41}U^0 + L_{42}W^0 + L_{43}\delta_{\gamma}^0 + L_{44}\tau_{\alpha\gamma}^0 \\ \sigma_{\gamma} &= L_{51}U^0 + L_{52}W^0 + L_{53}\delta_{\gamma}^0 + L_{54}\tau_{\alpha\gamma}^0 \end{aligned}\right\} \qquad (4)$$

In this case, the relations(4) are significantly simplified, since $\sigma_{\gamma}^0 = \tau_{\alpha\gamma}^0 = 0$

Then get:

$$\left.\begin{aligned} \sigma_{\gamma} &= L_{31}U^0 + L_{32}W^0 \\ \tau_{\alpha\gamma} &= L_{41}U^0 + L_{42}W^0 \\ \sigma_{\gamma} &= L_{51}U^0 + L_{52}W^0 \end{aligned}\right\}$$

(5)

Substituting(5) in the system (3), we get:

$$\left.\begin{aligned} \sqrt{1+\left(\frac{dh}{d\alpha}\right)^2}\, P_{\alpha} &= \frac{dh}{d\alpha}\left(L_{51}^{(h)}U^0 + L_{52}^{(h)}W^0\right) - \left(L_{41}^{(h)}U^0 + L_{42}^{(h)}W^0\right) \\ \sqrt{1+\left(\frac{dh}{d\alpha}\right)^2}\, P_{\gamma} &= \frac{dh}{d\alpha}\left(L_{41}^{(h)}U^0 + L_{42}^{(h)}W^0\right) - \left(L_{31}^{(h)}U^0 + L_{32}^{(h)}W^0\right) \end{aligned}\right] \qquad (6)$$

In contrast to similar dependencies for shells of constant thickness, here the coefficients in the operators Lmn(m=3,4,5; n=1,2) amL ...are functions of α.

In this regard, when performing the operator multiplication operation, the commutativity property is not observed, that is, for example: $L_{41}^{(h)} . L_{42}^{(h)} \neq L_{42}^{(h)} L_{41}^{(h)}$.

Therefore, for problems of this class, it is impossible to introduce a resolving function in the same way as it was done in the case of shells of constant thickness. However, for some particular problems, such a function can be found. If we restrict ourselves to the first terms in the operators, then due to the linear dependence of some coefficients in the Lmn operators, we can find a resolving function for the shell with free ends.

In this case, the Lmn operators will look like:

$$\left.\begin{aligned} I_{31} &= a_{3(1)}\frac{d}{d\alpha} \quad I_{32} = C_3(0) \quad I_{41} = a_4(2)\frac{d^2}{d\alpha^2} \\ I_{42} &= C_4(1)\frac{d}{d\alpha} \quad I_{51} = a_5(1)\frac{d}{d\alpha} \quad I_{52} = C_5(0) \end{aligned}\right\} \qquad (7)$$

Here, the following relations exist between the coefficients am[i], and cm[i] :

$$\left.\begin{aligned} a_{3(1)} &= vC_3(0) \quad C_3(0) = \frac{h(h+2)}{2(1+h)^2} \quad C_4(1) = va_4(2) \\ a_4(2) &= -\frac{h(h+2)}{2(1+h)} \quad C_5(0) = va_5(1) \quad a_5(1) = v \end{aligned}\right\} \qquad (8)$$

Taking into account these relations, the system(6) is converted to the form:

$$\left.\begin{array}{l}\sqrt{1+(\frac{dh}{d\alpha})^2}\,P_{\alpha}=\frac{dh}{d\alpha}\left(\frac{du^0}{d\alpha}-vw^0\right)-a_4(2)\left(\frac{d^2u^0}{d\alpha^2}-v\frac{dw^0}{d\alpha}\right)\\ \sqrt{1+(\frac{dh}{d\alpha})^2}\,P_{\gamma}=\frac{dh}{d\alpha}\left(\frac{d^2u^0}{d\alpha^2}-v\frac{dw^0}{d\alpha}\right)a_4(2)-C_3(0)\left(v\frac{du^0}{d\alpha}-w^0\right)\end{array}\right] \quad (9)$$

Let us consider the case when the forces acting on the external surface have only a vertical component, i.e., let us put P_ (∝=) 0. To find this solution, assuming in the system(9) $P_{\propto}$ and $W^0=\varphi$ $\frac{du^0}{d\propto}=-v\varphi$, we identically satisfy the first equation.

The second equation will take the form:

$$\sqrt{1+(\frac{dh}{d\alpha})^2}P_{\gamma}=(1-v^2)\bar{C}_3(0)\varphi \quad (10)$$

$$\varphi=-\frac{P_{\gamma}}{1-v^2}\frac{\sqrt{1+(\frac{dh}{d\alpha})^2}}{C_3(0)} \quad (11)$$

Accordingly, the initial function W^0 and the first derivative of the initial function $\frac{du^0}{d\propto}$ are determined by the following formulas:

$$\left.\begin{array}{l}\frac{du^0}{d\propto}=-\frac{vP_{\gamma}}{1-v^2}\frac{\sqrt{1+(\frac{dh}{d\alpha})^2}}{C_3(0)}\\ W^0=\frac{P_{\gamma}}{1-v^2}\frac{\sqrt{1+(\frac{dh}{d\alpha})^2}}{C_3(0)}\end{array}\right] \quad (12)$$

Substituting expressions (12) and (7) in the formulas of the main relations of the MNF, we obtain the final expressions for the values that characterize the VAT:

$$\left.\begin{array}{c}W(\propto,\gamma)=(1+\gamma)\varphi-(1+v)\frac{\gamma(\gamma+2)}{2(1+\gamma)}\varphi\\ \frac{\sigma_{\gamma}(\alpha,\gamma)}{E}=(1-v^2)\frac{\gamma(\gamma+2)}{2(1+\gamma)^2}\varphi\\ \frac{\sigma_{\gamma}(\alpha,\gamma)}{E}==(1-v^2)(1+\frac{1}{(1+\gamma)2})\varphi\\ \sigma_{\alpha}=\tau_{\alpha\gamma}=0\end{array}\right]$$

(13)

In conclusion, we present calculation formulas for some special cases of the shape of the outer surface of the shell.

1. A shell whose relative thickness is a linear function of the axial coordinate α, i.e. $h(\alpha)=h(1-k\propto)$ where K=tgθ is the tangent of the angle of inclination of the outer side surface relative to the axial coordinate α (figure 2).

In this case:

$$\frac{dh}{d\alpha}=-hk,\ P_{\gamma}=-\frac{P}{E},\ P_{\alpha}=0$$

$$\varphi=-\frac{P}{E(1-v^2)}\sqrt{1+h^2k^2}\,\frac{2(1+h(\propto))^2}{h(\propto)(h(\propto)+2)}$$

For the desired functions, we have:

$$W(\propto,\gamma)=\frac{P}{E(1-v^2)}\sqrt{1+h^2k^2}\left(1+\gamma-(1+v)\frac{\gamma(\gamma+2)}{2(1+\gamma)}\right)\frac{2(1+h(\alpha))}{(h(\alpha)h(\alpha)+2)}$$

$$\sigma_{\gamma}(\alpha,\gamma)==-P\sqrt{1+h^2k^2}\,\frac{\gamma(\gamma+2)}{(1+\gamma)^2}\frac{(1+h(\propto))^2}{h(\propto)(h(\propto)+2)}$$

$$\sigma_{\beta}(\alpha,\gamma)=-P\sqrt{1+h^2k^2}\left[1+\frac{1}{(1+\gamma)^2}\right]\frac{(1+h(\alpha))^2}{h(\alpha)(h(\alpha)+2)}$$

$$\sigma_{\alpha}(\alpha,\gamma)=\tau_{\alpha,\gamma}(\alpha,\gamma)=0$$

2. A shell whose relative thickness varies according to the hyperbolic law, i.e. $h(\alpha)=\frac{1}{1+\alpha}$ (Fig.3).

Similarly, we define:

$$\frac{dh}{d\alpha}=-\frac{h}{(1+\alpha)^2},\ P_{\gamma}=-\frac{P}{E},\ P_{\alpha}=0$$

$$\varphi=-\frac{P}{E(1-v^2)}\sqrt{1+\frac{h^2}{(1+\alpha)^4}}\,\frac{2(1+h(\propto))^2}{h(\propto)(h(\propto)+2)}$$

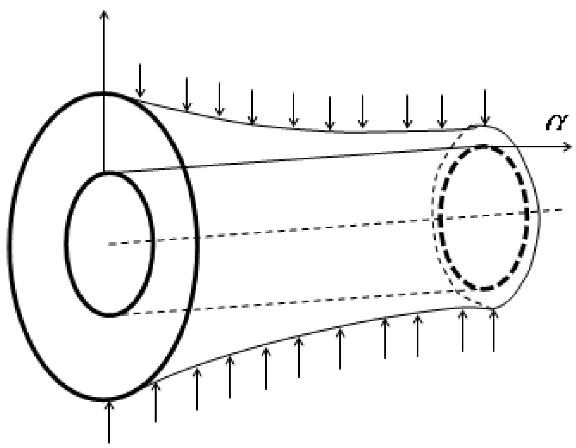


Figure 3

The desired functions are determined by relations(13) after substituting the expression for φ in them.

3. Shell, the thickness of which varies according to a parabolic law, i.e.

$$h(\alpha) = h(1 - k \propto^2)$$

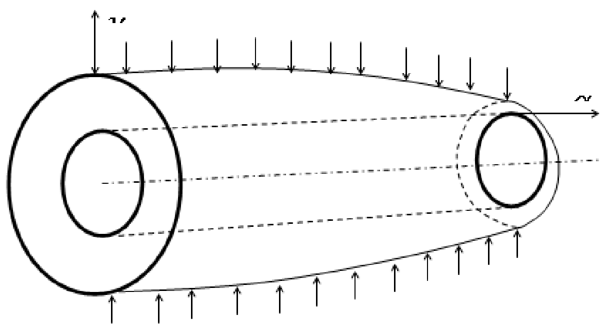


Figure 4

$$\frac{dh}{d\alpha} = -2hk \propto, \; P_\gamma = -\frac{P}{E}, \; P_\alpha = 0$$

$$\varphi = -\frac{P}{E(1-\nu^2)} \sqrt{1 + 4h^2k^2 \propto^2} \; \frac{2(1+h(\propto))^2}{h(\propto)(h(\propto)+2)}$$

As in the previous cases. σ_α, σ_β, and the displacement W are defined by relations (13) with the expression support φ.

Note that for a constant thickness $h(\propto) = Const$, all the results obtained coincide with the known lame relations.

When using the results obtained, it should be borne in mind that the given calculation formulas are approximate. The simplicity of the obtained dependencies makes it possible to use them for estimated preliminary calculations.

References

1. Agarev V. A. method of initial functions for two-dimensional boundary value problems of elasticity theory. Kiev, Publishing house of the Ukrainian SSR, 1963, 203 p.
2. Vlasov V. V. Method of initial functions in the plane problem of elasticity theory. –WPI. higher educational. Construction and architecture,1958, no. 2.
3. Gaffarov Kh. R. Improvement of the technological process and justification of the parameters of tools for decompression of the sub-arable soil layer in the cotton growing zone: abstract of dis. ... candidate of technical Sciences: 05.20.01. - Yangiyul, 1993.
4. .Ivengar K.T.S.R. Method of Initial Functions in the Analysis of thick circular Plates Nuklear Tngineering and Design. Vol.36.No.3, Mar,1976, PP. 341-354.
5. Galileev M. D., Davronov O. D. Method of initial functions in the theory of circular shells. - DEP. no. 1913, TSINIS, 1980, p. 24.
6. Iyengar K. T. S. R. Method of Initial Functions in the Analysis of thick circular Plate Nuclear Engineering and Design. Vol.36. No. 3, Mar, 1976, pp. 341-354.
7. A..E..Belkin, S. S. Gavryushin. - Moscow: Publishing house of the Bauman Moscow state technical University, 2008. - 231 p.
8. Vlasov, V. Z. General theory of shells and its application in technology – M., 1949. – 766 p.
9. Sidorov, V. N. Lectures on strength of materials and theory of elasticity – M.: RIC Gene. staff of the Russian Federation, 2002. – 352 p.
10. Gorodetsky, A. S. Computer models of structures– M: Publishing house ASV, 2009. – 357 p.

11. Karpilovsky, V. S. Methods of designing finite elements. - Kiev,. 1980. - 50 p. DEP. in Ukrniinti 23.06.80.
12. Argov, M. I. the Theory of perfectly plastic solids and structures – M.: Nauka, 1978. – 330 p.
13. Kolkunov, N. In. Bases of calculation of elastic shells – M.: Higher school, 1972. – 296 p.
14. Monks, V. A. bearing capacity of shallow arches .- Structural mechanics and calculation of structures. – 1976. – No. 5. – P. 30-35.
15. Monks, V. A. Dynamics of elastoplastic systems in phase space– Penza: PGWC, 2006. – 352 p.
16. Monks, V. A. Kinematics. Lectures on theoretical mechanics. Part 2. - Penza: PGUAS, 2014. - 124 p.
17. Monakhov, I. A. Calculation of rigid-plastic intersecting cylindrical shells that are under the action of internal pressure-DEP. V TSINIS, NTL, sec. B, issue 12, 1977.
18. Ogibalov, P. M. Plates and shells-Moscow: MSU, 1968 – - 695 p.
19. Software complex SCAD Office 11.3, 2009.
20. Smirnov, A. F. Calculation of structures using computers-M.: stroizdat, 1964. - 380 p.
21. Timoshenko, S. P. Theory of plates and shells-M.: stroizdat, 1977 – - 686 p. 231
22. Smith I.M. end Griffiths D.V. Programming the Finite Element Method. John Wiley end Sons. Chichester, Toronto. 1988. - 469 pp.
23. Zienkiewicz, O.C., Taylor, R.L. The Finite Element Method. Vol. 1, 2. Oxford. 2000.- 459 p.
24. Charkov, V. D. Determination of the load-bearing capacity of axisymmetric shells by linear programming. News Of Universities. Mechanical engineering. – 1969. – №5.
25. Tsetyrkin, I. E. Influence of the side beam height on internal forces in a cylindrical shell .Construction mechanics: sat. Tr. Miita. - M., 1971. - Issue 364. - Pp. 68-77.
26. Smith I. M. end Griffiths D. V. Programming the Finite Element Method. John Wiley and Sons. Chichester, Toronto. 1988. - 469 PP.
27. Zienkiewicz, O. C., Taylor, R. L. The Finite Element Method. Vol. 1, 2. Oxford. 2000.- 459 p.
28. V. P. Bazlov // Construction mechanics and calculation of structures. - 1977. - no. 5. - P. 26-29.
29. Volkov A. N. Statics of thick shells. The author's abstract Diss.on the map. Uch. step. Doct. tech.nauk, M., 1974, p. 33.