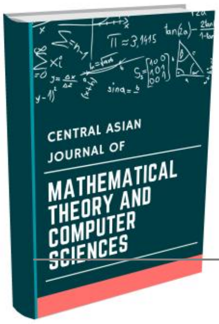

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ON THE RESULTS OF FUZZY METRIC ALGEBRA ON THE FUZZY FIELD

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Abstract

In this research, we dealt with the concept of fuzzy metric algebra on the fuzzy field, and closure in fuzzy metric algebra on the fuzzy field and we prove some characteristics of this subject.

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1. Introduction: It has been studied fuzzy fields and fuzzy linear by S. Nanda in 1986 [7], and this researcher also studied fuzzy algebras over fuzzy fields in 1990 [8]. After that, the researchers G. Wenxiang and L. Tu in 1993 [5] modified the definition of fuzzy algebras over fuzzy fields. Hence, many researchers carried out studies until the researchers G.Gebray and B. KrishnaReddy in 2014 [4] reached a definition of fuzzy metric on fuzzy linear spaces. We will know fuzzy metric algebra over the fuzzy field and We show some important theorems.

2. Preliminaries

Definition{2.1}:-[2] Let $\circledast$:[0,1]×[0,1]→[0,1] is binary operation is defined a **continuous t-norm** if (([0,1]) ,$\circledast$) is a topological monoid with unit 1 ∋z$\circledast$e≤v$\circledast$n whenever v≥z and n≥e z,e,v,n ∈ [0,1] .

Definition{2.2}:-[3] Let (H, J) be fuzzy linear spaces on the fuzzy field (W, Q). The function D_ρ:J×J⟶Q is defined as a **fuzzy metric space** ((H, J), D_ρ) over (W, Q) if satisfying the following conditions:

$\boxed{m1}$ W$\left(D_\rho(l,u)\right) \geq \min\{H(l),H(u)\}$,$\forall l\,,u \in J$.

$\boxed{m2}$ $D_\rho(l\,,u) \geq 0$ for all $l,u \in J$.

$$\boxed{m3}\; D_\rho(l,u) = 0 \iff l = u\,.$$

$\boxed{m4}$ $D_\rho(l\,,u) = D_\rho(u\,,l)$ for all $l\,,u \in J$.

$\boxed{m5}$ $D_\rho(l\,,u) \leq D_\rho(l\,,q) + D_\rho(q\,,u)$ for all$l\,,u,q \in J$

Definition {2.3}:-[4] Let (W, Q) be a fuzzy field in Q, J be linear space on Q, and let (H, J) be a fuzzy linear space on (W, Q). The norm on (H, J) is a function, ‖·‖: J⟶Q satisfies the following conditions:

$\boxed{m1}$ W(‖ l ‖) $\geq H(l)$ for all $l \in J$.

$\boxed{m2}$ ‖ l ‖ ≥ 0 for all $l \in J$.

$\boxed{m3}$ ‖ l ‖ $= 0 \Leftrightarrow$ if $l = 0$.

$\boxed{m4}$ ‖ λl ‖ $= |\lambda|$ ‖ l ‖ for all $\lambda \in Q$ and $l \in J$.

$\boxed{m5}$ ‖ $l + u$ ‖ $\leq$ ‖ l ‖ $+$ ‖ u ‖ for all $l, u \in J$.

The tuple $(H, J, \|\cdot\|)$is called **a fuzzy normed space on a fuzzy field**.

3. ON RESULTS FUZZY METRIC ALGEBRA ON THE FUZZY FIELD

Definition {3.1}:- Let (W, Q) be a fuzzy field in Q. In algebra J there is a fuzzy set which is H on Q then ((H, J), D_ρ) is **fuzzy metric algebra on (W, Q)** if :

$\boxed{m1}$ J is a fuzzy algebra.

$\boxed{m2}$ $((H, J),\ D_\rho)$ is a fuzzy metric on (H, J).

$\boxed{m3}$ $D_\rho(l\,, u) \leq D_\rho(l\,, q) \odot D_\rho(q\,, u)\,, \forall\, l\,, u\,, q\ \in\ J.$

Definition {3.2}:-[6] Let (W, Q) be a fuzzy field in Q. In algebra J there is a fuzzy set which is H on Q then ((H, J),‖·‖) is a **fuzzy normed algebra over (W, Q)** if **:**

$\boxed{m1}$ (J) is a fuzzy algebra.

$\boxed{m2}$ $((H, J), D_\rho)$ is a fuzzy norm on (H, J).

$\boxed{m3}$ ‖ $l \cdot u$ ‖ $\leq$ ‖ l ‖ $\odot$ ‖ u ‖ $, \forall\, l\,, u\,, q\ \in\ J.$

Theorem {3.3}:- Let $(H, J, \|\cdot\|)$ over(W, Q), and $D_\rho(l, u) =\| l \ominus u \|$. then $((H, J),\ D_\rho)$ is $fuzzy\ metric\ algebra.$

Proof:

$\boxed{m1}$ (J) is a fuzzy algebra. This is proven in the source [1]

$\boxed{m2}$ $((H, J),\ D_\rho)$ is the fuzzy norm on (H, J) $then$ $((H, J), D_\rho)$ is the fuzzy norm on (H, J). This is proven in the source [2]

$\boxed{m3}$ $\| l \cdot u \| \leq \| (l \ominus q) \odot (q \ominus u) \| \leq \| l \ominus q \| \odot \| q \ominus u \|$ $then$

$D_\rho(l\,, u) \leq D_\rho(l\,, q) \odot D_\rho(q\,, u)\,, \forall\, l\,, u\,, q\ \in\ J.$

Definition{3.4}:- Let $((H, J),\ D_\rho)$ is fuzzy metric algebra for each $c_0 \in J\,, 0 < \omega$ the ***open ball*** $G_\omega(c_0) in\, J$ radians and center at c_0 then

$$G_\omega(c_0) = \{l\ \in J : \omega > D_\rho(l\,, c_0), \mathrm{W}\left(D_\rho(l, c_0)\right) \geq \min\{\ H(l), H(c_0)\}.$$

As for definition ***closed ball*** $\overline{G_\omega(c_0)}$ in J of radians and center at c_0 then

$$\overline{G_\omega(c_0)}\ = \{l\ \in J : \omega \geq D_\rho(l, c_0), \mathrm{W}\left(D_\rho(l, c_0)\right) \geq \min\{\ H(l), H(c_0)\}.$$

Definition {3.5}:- Let $((H, J),\ D_\rho))$ over (W, Q) and $\mathrm{N} \sqsubseteq J$. N is said to be ***an open set*** in J if $\forall\, l \in \mathrm{N}, \exists\ 0 < \omega, \exists\ G_\omega(l) \sqsubset$ N. And if N^c is an open set in J said N is to be ***a closed set*** in J.

Theorem {3.6}:- Let $((H, J),\ D_\rho)$ over (W, Q). Then

$\boxed{m1}$ $\forall G_\omega(c_0)$ball will be an open set.

$\boxed{m2}$ $\forall\overline{G_\omega(c_0)}$ will be a closed set.

Poof:

Let $((H,J),D_\rho)$ be a fuzzy metric algebra and $c_0 \in J, l < \omega$

$\boxed{m1}$ We have to prove $G_\omega(c_0)$ is an open set

Suppose that $l \in G_\omega(c_0)$ And so van $\omega > D_\rho(l, c_0), \omega - D_\rho(l, c_0) > 0$.

So we will take $\omega_1 = \omega - D_\rho(l, c_0), 0 < \omega_1$ So we have to prove

$G_{\omega_1}(l) \sqsubset G_\omega(c_0)$.

Let $u \in G_{\omega_1}(l)$ we get $\omega_1 > D_\rho(u,l)$,$\therefore \omega - D_\rho(l, c_0) > D_\rho(u,l)$ hence

$\omega > D_\rho(l, c_0) - D_\rho(u, l)$

$\because D_\rho(u, l) + D_\rho(l, c_0) \geq D_\rho(u, c_0), \therefore D_\rho(u,c_0) < \omega$.

$\mathcal{W}\left(D_\rho(u,c_0)\right) \geq H(u - c_0) \geq \min\{H(u), H(c_0)\}$. Hence $u \in G_\omega(c_0)$

From this, we conclude that $G_\omega(c_0)$ *is an open set*.

$\boxed{m2}$ We have to prove $\overline{G_\omega(c_0)}$ is a closed set.

Suppose that $\mathrm{N} = (\overline{G_\omega}(c_0))^c$ And so van $\overline{G_\omega(c_0)} = \{l \in J : \omega \geq D_\rho(l, c_0)$, $\mathcal{W}\left(D_\rho(l, c_0)\right) \geq \min\{H(l), H(c_0)\}$.

$\mathrm{N} = \{l \in J : \omega > D_\rho(l, c_0), \mathrm{W}\left(D_\rho(l, c_0)\right) \geq \min\{H(l), H(c_0)\}$.

Let $l \in \mathrm{N}, \therefore \omega > D_\rho(l, c_0)$, but

$\omega_2 = D_\rho(l, c_0) - \omega, 0 < \omega_2$, So we have to prove

$G_{\omega_2}(l) \sqsubset \mathrm{N}, \omega < D_\rho(l, c_0) - D_\rho(u, l)$ hence

$\because D_\rho(l, c_0) - D_\rho(u, l) \leq D_\rho(u, c_0), \therefore D_\rho(u, c_0) < \omega$.

$\mathrm{W}\left(D_\rho(u,c_0)\right) \geq H(u - c_0) \geq \min\{H(u), H(c_0)\}$. Hence $u \in \mathrm{N}, G_{\omega_2}(l) \sqsubset \mathrm{N}$

Then N is an open set. From this, we conclude that $\overline{G_\omega(c_0)} = \mathrm{N}^c$ *is an open set*.

Definition {3.7}:- Let $((H,J), D_\rho)$ be fuzzy metric algebra, $\mathrm{N} \sqsubseteq \mathrm{J}$. We say about a limit point to set N if the point $l \in J$ *and for each* $0 < \omega$ There is $u \in \mathrm{N}$ such that $l \neq u$, *if* $\omega > D_\rho(l,u), \mathrm{W}\left(D_\rho(l,u)\right) \geq \min\{H(l), H(u)\}$.

The set all limit points to the set N is defined (***Derived***) the to set N, denoted by N'.

$\mathrm{N}' = \{l \in J : \forall\omega > 0, \exists u \in \mathrm{N}, \ni u \neq l$ *if* $D_\rho(l, u) < \omega$,

$\mathrm{W}\left(D_\rho(l, u)\right) \geq \min\{H(l), H(u)\}$. Or We say about a limit point to set M if G_x opens set in J and if $u \in G_x$, such that $\mathrm{N} \sqcap \{G_x/\{l\}\} \neq \emptyset$. The set all limit points to the set N is defined (*Derived*) to the set N, denoted by N'.

Definition {3.8}:- Let $((H,J), D_\rho)$ be fuzzy metric algebra, $\mathrm{N} \sqsubseteq \mathrm{J}$. We say about the point to set N is a closure if the point $l \in J$ *and for each* $0 < \omega$ There is $u \in \mathrm{N}$ such that $\omega > D_\rho(l,u)$, $\mathrm{W}\left(D_\rho(l,u)\right) \geq \min\{H(l), H(u)\}$.

The set whose members are all the closure points of set N is defined **($Closure$)** of set N and denoted by $\overline{\mathrm{N}}$.

$\overline{\mathrm{N}} = \{l \in J : \forall\omega > 0 , \exists\, u \in \mathrm{N} , \ni D_\rho(l , u) < \omega, \mathrm{W}\left(D_\rho(l , u)\right) \geq \min\{ H(l), H(u)\}$.

Theorem {3.9}:- Let $((H, J), D_\rho)$ fuzzy metric algebra and $\mathrm{N} \sqsubseteq J$. Then

$\boxed{m1}\ \mathrm{N}' \sqsubset \overline{\mathrm{N}}\ \boxed{m2}\ \overline{\mathrm{N}} = \mathrm{N} \sqcup \mathrm{N}'$

Proof: Suppose that $l \in \mathrm{N}', for\ each\ 0 < \omega$ There is $u \in \mathrm{N}$ such that $u \neq l$ and $D_\rho(l , u) < \omega$, $\mathrm{W}\left(D_\rho(l , u)\right) \geq H(l - u) \geq \min\{ H(l) , H(u)\}$, $\forall\ 0 < \omega , \exists\ u \in \mathrm{N}$ and $D_\rho(l , u) < \omega$, $\mathrm{W}\left(D_\rho(l , u)\right) \geq H(l - u) \geq \min\{ H(l) , H(u)\}$ then $l \in \overline{\mathrm{N}}$

Hence $\mathrm{N}' \sqsubset \overline{\mathrm{N}}$

$\boxed{m2}\ \because\ \mathrm{N} \sqsubset \overline{\mathrm{N}}, \mathrm{N}' \sqsubset \overline{\mathrm{N}}$ hence $\mathrm{N} \sqcup \mathrm{N}' \sqsubset \overline{\mathrm{N}}$.

Now let that $l \in \overline{\mathrm{N}}$.

$IF\ l \in \mathrm{N}$ then $l \in \mathrm{N} \sqcup \mathrm{N}'$ then $\overline{\mathrm{N}} \sqsubset \mathrm{N} \sqcup \mathrm{N}', if\ l \notin \mathrm{N}$, $\because l \in \overline{\mathrm{N}}, \forall\omega > 0, \exists\, u \in \mathrm{N}$, $D_\rho(l, u) < \omega$, $\mathrm{W}\left(D_\rho(l, u)\right) \geq H(l - u) \geq \min\{ H(l), H(u)\}$

$\because l \notin \mathrm{N}, \therefore u \neq l, \exists\, u \in \mathrm{N}'$ and $l \in \mathrm{N} \sqcup \mathrm{N}'$, then $\overline{\mathrm{N}} \sqsubset \mathrm{N} \sqcup \mathrm{N}'$.

There for $\overline{\mathrm{N}} = \mathrm{N} \sqcup \mathrm{N}'$.

Theorem {3.10}:- Let $(H, J, \|\cdot\|)$ fuzzy normed algebra and $\mathrm{N} \sqsubseteq J$. Hence

$\boxed{m1}\ \mathrm{N}' \sqsubset \overline{\mathrm{N}}$

$$\boxed{m2}\ \overline{\mathrm{N}} = \mathrm{N} \sqcup \mathrm{N}'$$

Proof: The same method of proving the theorem {3.9}. B. References

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