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Methods of Determining Points with Rational Coordinate to Line

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Abstract

The article discusses the definition of rational coordinate points related to a curve in a plane, as well as their applications in number theory and other areas of mathematics.

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It is known that the solution of various complex equations in mathematics caused the introduction of new ideas into the theory of numbers, like all areas of mathematics. Among these ideas, the ideas of geometry and topology can be listed as the main ones. According to N. Burbaki: "Classical geometry ... has reached the level of a universal language that takes into account the subtleties and conveniences of general mathematical sciences." In addition, geometry appears not only as a universal language, but also as a science that offers unique methods for solving problems.

Let's consider an example of geometric language application. For example,

$$x^n + y^n = z^n$$

Let's look at Fermat's equation. Dividing both sides of the equation by z^n and denoting x/z by u and y/z by v, we get this equation.

$$u^n + v^n = 1$$

Each (x, y, z) solution of Fermat's equation $(z \neq 0)$ corresponds to a solution of the equation $u^n + v^n = 1$ consisting of rational numbers.

Let's look at the line defined by this equation using the points (u, v) in the plane. The integer solution of Fermat's equation is the rational coordinates of the resulting line. It is also visible from the opposite side: the rational coordinate points of the resulting line (when saved from the denominator) are all solutions of Fermat's equation.

Thus, turning Fermat's problem into a geometric language comes to the problem of finding points with rational coordinates of the line $u^n + v^n = 1$.

(We did not consider the $z = 0$ solution when we divided Fermat's equation by z^n, so this solution can be studied separately).

We consider below the problem of finding the rational points of a curve in two ways. Let's look at this equation.

$$x^2 + y^2 = z^2$$

The solutions of this equation consist of a triplet of natural numbers called the Pythagorean triplet. It is known that

$$x^2 + y^2 = z^2$$

the curve corresponding to the equation consists of a circle $u^2 + v^2 = 1$. Points $(3/5;\ 4/5), (5/13;\ 12/13)$ correspond to the numbers $(3,4,5), (5,12,13)$ belonging to the Pythagorean triad.

When working with some smooth curves, it is convenient to work with their parametric equations. That is, here for each value of the parameter t there corresponds a point on the curve $(f(t), g(t))$ and for each point $(x_0; y_0)$ the corresponding value t_0 of the parameter t is found, where $x_0 = f(t_0);\ y_0 = g(t_0)$.

For example, if the point P on the circle $(u; v)$ corresponds to an angle α, then

$$u = cos\alpha, \quad v = sin\alpha$$

parametric equations can be given using $sin\alpha$ and $cos\alpha$. Parametric equations of the circle can also be given in a different form.

We will do the construction in the 2nd drawing. From the similarity of triangles OAt_0 and QPu_0

$$u = \frac{1-t^2}{1+t^2}, \quad v = \frac{2t}{1+t^2}$$

equations are derived (using the first equation, the second equation is derived by replacing the parameter t with $tg\dfrac{\alpha}{2}$).

It can be seen that when u and v coordinates are represented by t, rational values of parameter t correspond to points with rational coordinates lying on a circle.

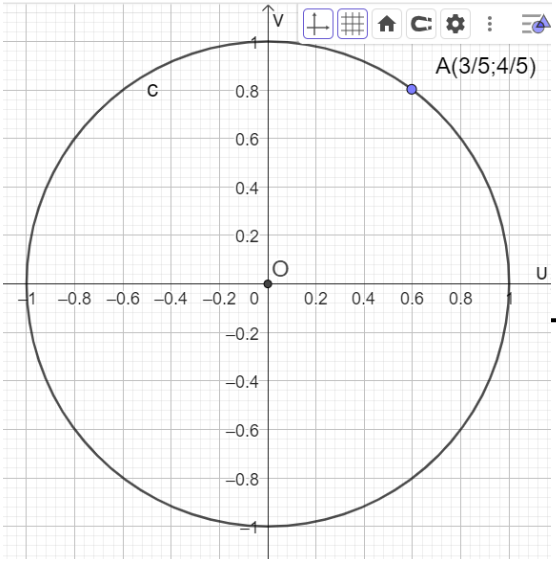

Fig. 1

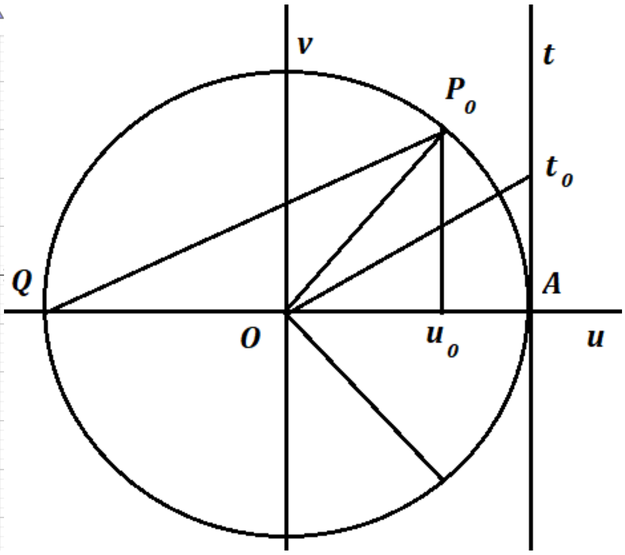

Fig. 2

Example 1. Prove that if the coordinates $(u; v)$ are a rational point of the circle, then there exists a unique rational-valued parameter t giving the values u and v.

Solution. Let's take one of the equations $u = \frac{1-t^2}{1+t^2}$, $v = \frac{2t}{1+t^2}$, for example the second one. In this equation, v is a rational number by convention. To find the parameter t, we create a quadratic equation of the form $vt^2 - 2t + v = 0$. The discriminant of the quadratic equation is $D = 4 - 4v^2 = 4(1 - v^2) = 4u^2$.

Since the discriminant is greater than zero, it is clear that the quadratic equation has two solutions, and at the same time, these solutions are rational solutions. Because the coefficients of the equation, at the same time, the number $\pm 2u$ are also rational numbers, and the fact that the set of rational numbers is closed with respect to the four ends means that t is a rational number.

From the considered problem, it became clear that there is a one-value correspondence between the rational points of the circle and the rational values of the parameter t. The algebraic representation of this compatibility is shown in the above formula, and the geometric representation is shown in diagram 2. The main conclusion is that the problem of finding rational points on a circle is solved by the parameter t.

Besides circles, many lines can be parameterized by fractional-rational functions. Such lines are called *rational lines*. Examples of such lines are all second-order lines.

Example 2. parameterize the line $y^2 = x^3 - x^2$ rationally.

Solution. An illustration of this line is given in Figure 3.

The image of this line consists of a distinct point O and a line whose branches are symmetrical to the Ox axis.

We pass all possible straight lines $y = tx$ through point O. It is convenient to implement the angle coefficient t using a line drawn $(1; 0)$ as in fig. 4.

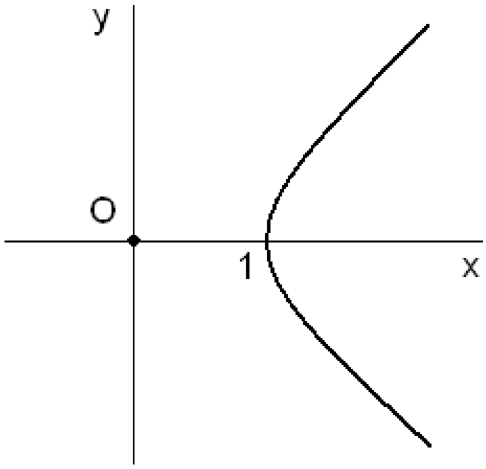

Fig. 3.

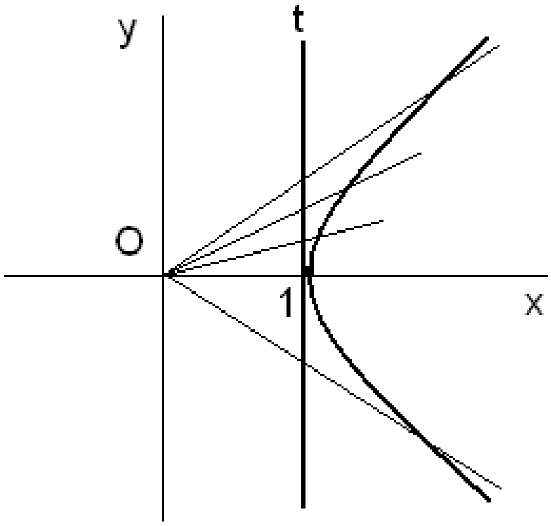

Fig. 4.

Substituting $y = tx$ into the equation of the curve, we reduce the resulting equation to x^2 (where O is the special point), and we obtain the parametric equation

$$\begin{cases} x = t^2 + 1 \\ y = t(t^2 + 1) \end{cases}.$$

Here, the points with rational coordinates on the curve $y^2 = x^3 - x^2$ correspond to the rational values of the variable t.

Thus, it is possible to establish a one-value correspondence with the rational coordinate points of the curve $y^2 = x^3 - x^2$ (except for the point $(0,0)$) and the straight line t. Figure 4 represents the geometrically central projection.

Similarly, it can be shown that the curve $y^2 = x^3 + x^2$ is parametrized.

$$\begin{cases} x = t^2 - 1 \\ y = t(t^2 - 1) \end{cases}$$

In conclusion, the problem of determining the rational coordinate points related to the line is important, and scientific research in this regard continues rapidly.

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