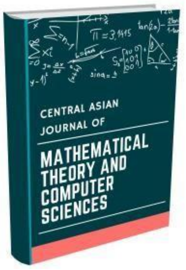

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Article

Methodological Approaches To Solving Complex Geometric Problems Of Pyramids

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Abstract: This study addresses the complexities of solving geometric problems involving pyramids, focusing on methodological approaches to enhance understanding and application in educational settings. Despite extensive research in geometric problem-solving, a significant knowledge gap remains in effectively teaching complex pyramid problems. Utilizing a blend of empirical and theoretical methods, including pedagogical experience analysis, teacher consultations, and interdisciplinary synthesis, the study developed practical tasks and control measurements. Findings reveal that a systematic approach to teaching pyramid geometry significantly improves student comprehension and problem-solving skills. The results underscore the importance of integrating diverse methodological strategies in mathematics education, offering implications for curriculum development and instructional practices.

Keywords: Pyramid, Geometric Figure, Triangle, Quadrilateral, Polygon, Tetrahedron, Distance

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1. Introduction

A pyramid is a polyhedron that consists of a flat polygon - the base of the pyramid, a point that does not lie in the plane of the base, the top of the pyramid and all segments connecting the top of the pyramid with the base points[1]. Figure 1 shows the SABCD pyramid, where ABCD is the base and S is the vertex. Triangles SAB, SBC, SCD, SDA are called side faces. Straight SA, SB, SC, SD are called pyramid side edges. The perpendicular SO dropped from the top to the base is called the height of the pyramid and is denoted by H.

The section of the pyramid that passes through the vertex and diagonal of the base is called the diagonal section of the pyramid. For example, the triangle ASC is the diagonal section of the pyramid[2].

A pyramid is called triangular, quadrilateral, etc., if its base is a triangle, quadrilateral, etc [30].

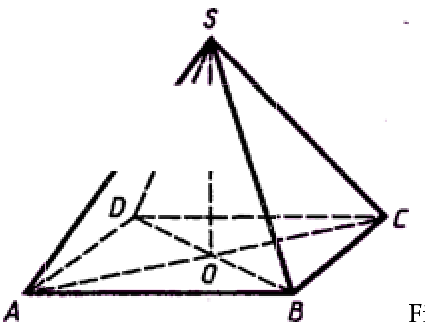


Fig. 1.

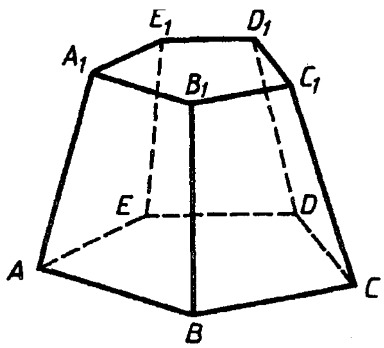


Fig. 2

A pyramid is called regular if its base is a regular polygon, and its height passes through the center of the base.

The side faces of a regular pyramid are isosceles triangles equal to each other. The height of the side face of a regular pyramid is called the apothema of the pyramid. A triangular pyramid is also called a tetrahedron. If all four faces of a tetrahedron are regular triangles, then the tetrahedron is also called regular. If the pyramid is crossed by a plane parallel to the base, then[3]:

- side ribs and height will be divided into proportional parts;
- in the section you get a polygon similar to the base;
-Section and base areas are treated as squares of their distances from the vertex.

If the pyramid is crossed by a plane parallel to the base, then a new polyhedron is obtained, which is called a truncated pyramid (Fig. 2). ABCDE Polygon - Bottom, Polygon, $A_1B_1C_1D_1E_1$ Top[4].

The side surface area of a regular pyramid is equal to half the product of the base perimeter by the apothem[5]. Area of side surface of regular truncated pyramid is equal to product of half sum of perimeters of its bases by apopheme. If all side edges in the pyramid are equal, then its vertex is projected to the center of the circumscribed circle near the base. If in the pyramid all dihedral angles at the base are equal, then the vertex is projected to the center of the circle inscribed at the base[28].

The purpose of the study is to identify the methodological features of the study of the topic "Solving complex problems of the pyramid[6]."

2. Materials and Methods

The methodology employed in this study on solving complex pyramid problems integrates both empirical and theoretical research methods to comprehensively address the research objectives. The initial phase involved a thorough study and generalization of existing pedagogical experiences and scholarly works related to pyramid geometry. Conversations with experienced mathematics teachers provided practical insights and reinforced the theoretical underpinnings[7]. This was followed by an interdisciplinary analysis that synthesized psychological, pedagogical, and methodological principles pertinent to the study of geometric problems involving pyramids[29].

The research also entailed a detailed analysis of current educational standards, curricula, textbooks, and control [27] and measurement materials used in final certification exams for mathematics and geometry. This helped identify common challenges and effective strategies in teaching and learning about pyramids. Practical tasks were designed to test the understanding and application of geometric principles in pyramid-related problems. For instance, problems involving calculating the height of the pyramid, side surface area, and angles formed with the base were systematically solved to illustrate various geometric concepts[8].

Additionally, control measurements were conducted to compare and generalize the results, ensuring the [26] robustness and reliability of the proposed solutions. These measurements were instrumental in refining the methodological approaches and validating the theoretical models proposed[9]. The results were then synthesized to develop a set of guidelines for educators, aimed at enhancing the teaching and learning of complex geometric problems involving pyramids. This comprehensive approach ensures that the study not only addresses the immediate research questions but also contributes to the broader field of mathematics education by providing a framework for solving complex pyramid problems[10].

3. Results and Discussion

Problem With Solutions

Task 1. The base of the pyramid is a rectangle with sides 3 and 4 m. Each edge of the pyramid is 13 m[23]. Find the height of the pyramid and the area of the side surface.

Solution. 1. The condition of the problem is satisfied by Figure 3[11].

2. Since all side edges are equal by condition, the vertex is projected to the center of the circle described near the base, that is, to the point O of intersection of the diagonals.

3. Therefore, the height of the pyramid is equal to the leg of the right triangle OSD, in which the other leg is equal to half the diagonal of the rectangle, and the hypotenuse is the side edge[12].

4. Let's find the diagonal of the rectangle $BD = \sqrt{3^2 + 4^2} = 5$.

5. Pyramid height $SO = \sqrt{13^2 - 2{,}5^2} = \sqrt{162{,}75}$.

6. To find the side surface area, you need to know the lengths of the apothemes SK and SM:

from the right triangle SKD we find: $SK = \sqrt{13^2 - 1{,}5^2} = \sqrt{166{,}75}$;

from the right triangle SKD we find: $SM = \sqrt{169 - 4} = \sqrt{165}$.

4. Let's find the diagonal of the rectangle BD = √ (3 ^ 2 + 4 ^ 2) = 5.

5. Pyramid height SO = √ (≤ 13 ≤ ^ 2 - ≤ 2.5 ≤ ^ 2) = √ 162.75.

6. To find the side surface area, you need to know the lengths of the apothemes SK and SM:

from the right triangle SKD we find: SK = √ (≈ 13 ' ^ 2 - ' 1,5 ' ^ 2) = √ 166.75;

from the right triangle SKD we find: SM = √ (169-4) = √ 165.

Find the side surface area[14]:

$S_{lat} 2S_{\Delta ASD} + 2S_{\Delta DSC} = 2 \cdot \frac{4\sqrt{165}}{2} + 2 \cdot \frac{3\sqrt{166{,}75}}{2}$,

$S_{lat} = 4\sqrt{165} + 3\sqrt{166{,}75}$ м^2.

Task 2. In a triangular pyramid, the sides of the base are 13, 14, 15, all side ribs make angles with the base equal to α. Find the height of the pyramid[21].

Solution. 1. Conditionally satisfies Figure 4[15].

2. Determine the position of point O relative to triangle ABC. Since the right triangles AOD, COD and BOD have equal leg OD and equal acute angle α, they are equal. Therefore, AO = OC = OB. This means that the point O is the center of the circumscribed near the base of the circle of radius AO.

3. We find the radius of the circumcircle from the formula R = abc/4S. For this purpose, we need to find S_ABC. Let's use Heron's formula:

$$S = \sqrt{21 \cdot (21 - 13)(21 - 14)(21 - 15)} = \sqrt{21 \cdot 8 \cdot 7 \cdot 6} = 84$$

4. The height of the pyramid will be found from a right triangle AOD with an acute angle α.

So, $tg\ \alpha = \frac{OD}{AO}$, или $AO \cdot tg\ \alpha = OD$. Следовательно, $OD = \frac{65}{8} tg\ \alpha$.

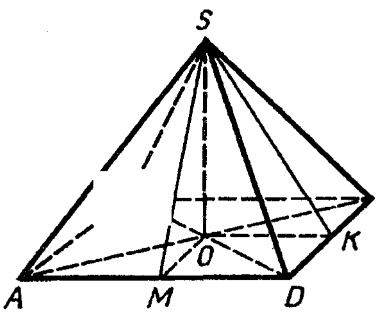


Рис. 3

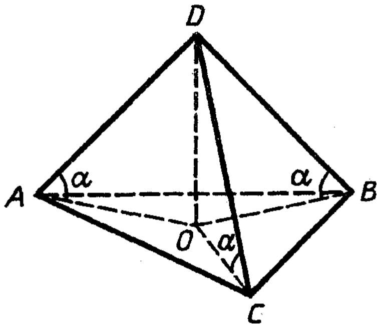


Рис. 4

Task 3. At the base of the triangular pyramid lies the isosceles triangle ABC, the angle C of the line. The lengths of the side edges of the pyramid are k, the length of the hypotenuse of the base is C. Find the angles that the side edges form with the base, and the dihedral angle with the edge CE (Fig. 5)[16].

Solution. 1. The equality of the side edges implies the equality of the angles that the side edges form with the plane of the base of the pyramid[17].

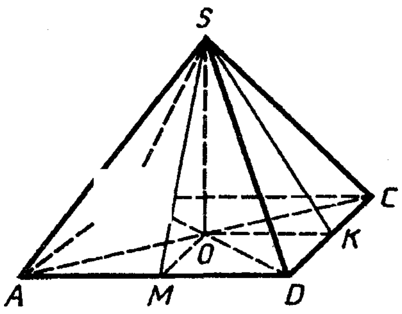
2. In this case, the height of the pyramid is projected to the center of the circle described around the base, i.e., in the middle of the hypotenuse of a right triangle ABC.

3. So, $EO\perp\ ABC\ \ и < EAO= < EBO=< ECO= x.$

4. To find these angles, consider the triangle AEO, где AE =k, $AO = \frac{c}{2}$.

Then $cos\, cos\ x\ = \frac{c}{2k}$, a $\frac{c}{2k}$.

5. To plot the linear angle of the dihedral angle at the EC edge, draw AD⊥EC and connect point D to B (Fig. 6). Since the triangles AEC and WEIGHT are equal, $BD\perp EC$ *and,* therefore, $< ADB$, equal α, and there is a desired linear angle[18].



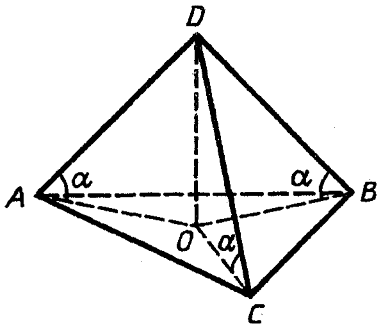


Рис. 3

Рис. 4

6. The ABD triangle is isosceles, DO is its median, and therefore both the height and the bisector. Therefore $< ODB = \frac{\alpha}{2}$.

7. In the triangle we have $OB = \frac{c}{2}$, and we find the side BD from the triangle BEC, the area of which, on the one hand $\frac{EC\cdot BD}{2}$, and on the other - $\frac{BC\cdot EM}{2}$. Thus, $\frac{EC\cdot BD}{2} = \frac{BC\cdot EM}{2}$, *или* $EC\cdot BD = BC\cdot EM$ (1) [19]

or equality (1) we have: $BD = \frac{BC\cdot EM}{ES} = \frac{BC\cdot EM}{k}$. (2)

a) BC find from a right isosceles triangle ABC hypotenuse equal to c: BC = $\frac{c}{\sqrt{2}}$

б) We will find from the EMC right triangle, where BM = MC= $\frac{c}{2\sqrt{2}}$.

Thus, $EM = \sqrt{k^2 - \frac{c^2}{8}} = \frac{\sqrt{8k^2 - c^2}}{2\sqrt{2}}$

8. Substitute in equality (2) the values of BC and EM, t.e[20].

$$BD = \frac{BC \cdot EM}{k} = \frac{\frac{c}{\sqrt{2}} \cdot \frac{\sqrt{8k^2-c^2}}{2\sqrt{2}}}{k} = \frac{c\sqrt{8k^2-c^2}}{4k}.$$

9. From right triangle DOB (Fig. 6)

$sin\ sin\ \frac{\alpha}{2} = \frac{OB}{BD} = \frac{c \cdot 4k}{2c\sqrt{8k^2-c^2}}$, from where $\alpha = 2\ arcsin\ arcsin\ \frac{2k}{\sqrt{8k^2-c^2}}$

4. Conclusion

The findings of this study demonstrate that a systematic and integrated approach to teaching complex geometric problems of pyramids significantly enhances student understanding and problem-solving abilities. By combining empirical and theoretical research methods, including pedagogical experience analysis and interdisciplinary synthesis, the study identified effective strategies for addressing the intricacies of pyramid geometry. The results suggest that incorporating these methodologies into the curriculum can lead to improved educational outcomes in mathematics. These insights have important implications for educators and curriculum developers, highlighting the need for comprehensive and diverse teaching methods. Future research should explore the application of these strategies in other geometric contexts and assess their long-term impact on student learning and performance.

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