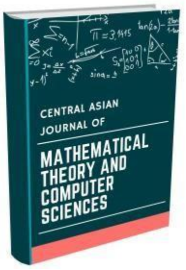

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Article

Construction of Graphs from the Complement of Discrete Topological Space

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Abstract: This paper introduces a new idea for constructing graphs from the complement of the discrete topological space, where many of the properties of the aforementioned topological statement were presented, When the number of elements in a non-empty set is two, the corresponding topological expression aligns with that of a null graph. In cases where the set contains three elements the topology statement is compatible with the circuit statement Moreover, we calculated the independent numbers, chromatic number, girth number, and clique number for Complement discrete topological space

Keywords: Topological graph, Complement graph, independence number, chromatic number, girth number, clique number, 2020 MSC: 05C69

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1. Introduction

A graph $G = (V, E)$ is an ordered pair of disjoint sets (V, E), where $V \neq \emptyset$ and E are a subset of unordered pairs of V. The elements $V = V(G)$ and $E = E(G)$ respectively are the vertices and edges of a graph G [1],[2]. In 1964, Ahlborn J. T.[3], a topology on a directed graph G (V, E) by a subset A of V is an open set if an edge does not exist from the set $V - A$ to the set A. Topology is one of the most active areas in mathematics. It is considered one of the basic specializations in mathematics due to the widespread use of this field by mathematicians in other sciences such as medicine, chemistry, etc. There are few studies in which topology is converted into a graph, despite its importance in demonstrating different properties and graphics, for more information about these topics in [4],[5],[6],[7]. The discrete topological space, denoted by (X, τ), consists of a non-empty set X and a family τ containing all subsets of X [8],[9]. In this study, we present a new perspective on discrete topological spaces by introducing a novel definition that transforms a specific type of topology into a graph, namely the complement of discrete topology. This graph is represented as $G\tau^{C}$= (V, E), where τ^{C}represents the closed topology derived from τ. The vertex set V ($G\tau^{C}$) comprises all subgroups of τ^{C} except for X and the empty set $\emptyset$. The edge set E ($G\tau^{C}$) includes edges between any two vertices if one is a subset of the other.

Several significant results have been established, including the characteristics of the graph $G\tau$ has an independent number set or the independent number of given graphs helps us to analyze social networks modeled as a graph, graphical information systems, and coding theory [10][11]. The coloring number of the graph indicates that no two adjacent vertices have the same color. The number of colors needed to color a chart is called the chromaticity number it is used in several aspects like team formation in expert networks, revealing the structure and function of networks, gene expression, and motif discovery in bioinformatics, computational chemistry [12] ,[13], [14]. Anomaly detection in complex networks, community search in social networks, traffic control problems, and social media analysis. Moreover, calculating the perimeter would be helpful in coding theory and software system testing and modeling.

2. Materials and Methods

The study aims to develop a method for constructing graphs from the complement of discrete topological spaces and analyze key properties like chromatic number, independent number, girth, and clique number.

A graph is created by representing subsets of the complement of a discrete topological space as vertices and using subset relationships to define edges. The study then calculates various graph properties that are important in fields such as network theory, coding theory, and computational biology.

3. Results

In this section, we introduce a novel method to construct a topological graph from the complement of a discrete topological space. Several key properties of these complement discrete topological graphs are identified and demonstrated.

Definition 2.1: Let X be a non-empty set and τ be a topology on X [15]. The complement of discrete topological graph denoted by $G\tau^{C}$ = (V, E) is a graph of the vertex set V = {M; M ∈ τ^{C} and M ≠ $\emptyset$, X} and the edge set E = {M, N; M ⊆ N or N ⊆ M}.

Definition 2.2: The maximum independent number (named independent) is the largest cardinality among all independent sets of a graph [16].While an independent set includes a set of vertices, no two vertices in the set are adjacent.

Definition 2.3: Coloring the graph indicates that no two adjacent vertices have the same color. The number of colors needed to color a chart is called the chromaticity number and is denoted by $\chi(G)$[17],[18].

Definition 2.4: The girth of a graph G is the length of the shortest cycle in G[19].

Definition 2.5: A clique in graph G is a complete subgraph of g and the order of the largest complete subgraph in G is the clique number denoted by $\omega(G)$[20].

Proposition 2.6: Let X $\neq \emptyset$ of order n and let $G\tau^{C}$ be the complement of a discrete topological space, then $G\tau^{C}$ is an empty graph when n equals 2.
Note: We refer to the vertices in the shapes mentioned in the article as follows:

$$\forall\, D_i = i\,;\, i = 1,2,3, \dots, n$$

Proof: According to Definition 2.1, for every pair of vertices in τ^{C}, either $\{r_1\}$ is not a subset of $\{r_2\}$ or $\{r_2\}$ is not a subset of $\{r_1\}$. Since $G\tau^{C}$ is a null graph, no such subset relationships hold between any pair of vertices, confirming that $G\tau^{C}$is indeed a null graph.

Fig. 1: The graph $G\tau^{C}$ is a null graph

Proposition 2.7: Let $X \neq \emptyset$ with cardinality n, and τ^{C} represent the complement topology on X. If n equals 2, then the independent number of $(G\tau^{C})$ = 2

Proof: by Proposition 2.2. According to the definition of an independent set, its vertices are not adjacent, and the proof is obtained.

Proposition 2.8: Let X Let $X \neq \emptyset$ with cardinality n, and τ^{C} denote the complement topology on X. If n equals 3, then the complement of the discrete topological space is isomorphic to the cycle graph (C_6

Proof: Let X = $\{D_1, D_2, D_3\}$, then τ = $\{\emptyset, X, \{D_1\}, \{D_2\}, \{D_3\}, \{D_1, D_2\}, \{D_1, D_3\}, \{D_2, D_3\}\}$,and τ^{C} = $\{X, \emptyset, \{D_2, D_3\}, \{D_1, D_3\}, \{D_1, D_2\}, \{D_3\}, \{D_2\}, \{D_1\}\}$ so $V(G\tau^{C})$ = $\{\{D_2, D_3\}, \{D_1, D_3\}, \{D_1, D_2\}, \{D_3\}, \{D_2\}, \{D_1\}\}$ Let M = $\{D_1\}$, N = $\{D_2\}$ represent vertices consisting of single elements. Since no single-element vertex is a subset of another, it follows that for all such vertices, neither $r_1\}$ is not a subset of $\{D_2\}$ or $\{D_2\}$ is not a subset of $\{D_1\}$ for all vertices of the single-element, by the definition of the complement of the discrete topological space, M is not adjacent to N. Hence, the vertices of the single element

$\{\{D_1\},\{D_2\},\{D_3\}\}$ are adjacent to the vertices of two elements that are required to be partial of them, and since $\{D_1\} \subseteq \{D_1, D_2\}$, therefore the two vertices are adjacent about For the vertices $\{D_1\} \subseteq \{D_1, D_3\}$, they are adjacent. Let us assume that u_1 and u_2 are vertices with two elements. Since $u_1 \not\subset u_2$ or $u_2 \not\subset u_1$ for any pair of vertices with two elements, u_1 is not an adjacent of u_2. As shown in Figure 2.

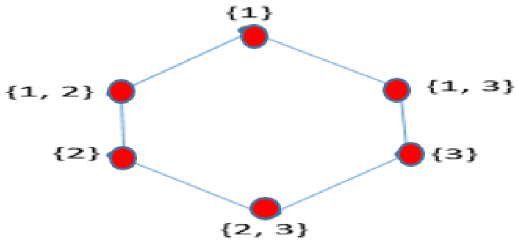


Fig. 2: graph $G\tau^C$ where X=3

Example 2.9: If $|X|$=4, then $\tau = \{\emptyset, X, \{D_1\},\{D_2\},\{D_3\},\{D_4\},\{D_1, D_2\},\{D_1, D_3\},\{D_1, D_4\},\{D_2, D_3\},\{D_2, D_4\},\{D_3, D_4\}\}$,and $\tau^C = \{X, \emptyset, \{D_3, D_4\}, \{D_2, D_4\},\{D_2, D_3\},\{D_1, D_4\},\{D_1, D_3\},\{D_1, D_2\},\{D_4\},\{D_3\},\{D_2\},\{D_1\}\}$ so $V(G\tau^C)=\{\{D_3, D_4\}, \{D_2, D_4\},\{D_2, D_3\},\{D_1, D_4\},\{D_1, D_3\},\{D_1, D_2\},\{D_4\},\{D_3\},\{D_2\},\{D_1\}\}$. As shown in Figure 3.

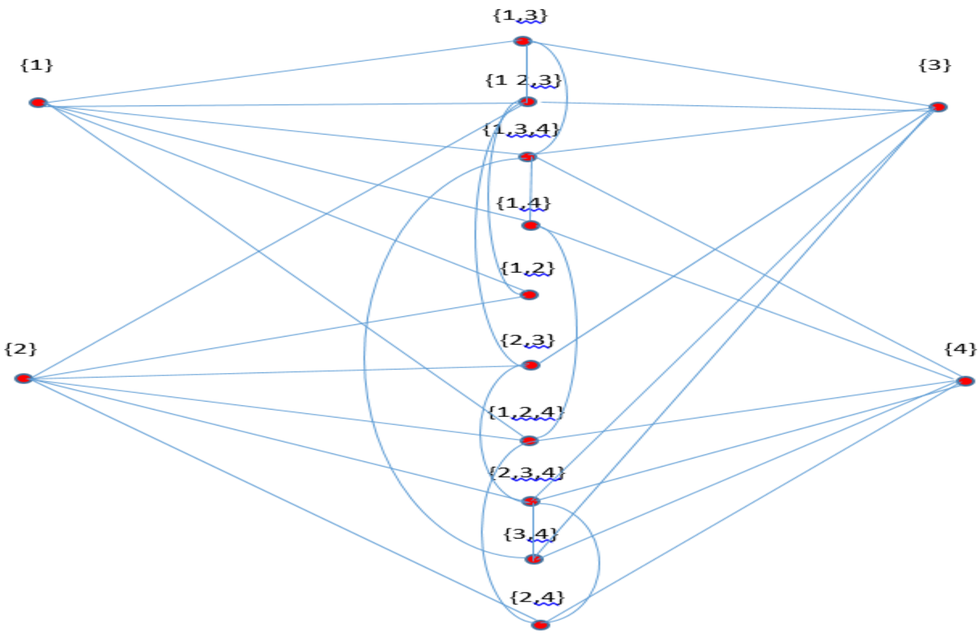


Fig. 3: The graph $G\tau^C$ where X=4

Example 2.10: If $|X|$=5, then

τ={$\emptyset$,X,{ D_1},{D_2},{ D_3},{D_4},{D_5},{D_1, D_2},{D_1, D_3},{D_1, D_4},{D_1, D_5},{D_2, D_3},{D_2, D_4},{D_2, D_5},{D_3, D_4},{D_3, D_5},{D_4, D_5} },and τ^C = { X , $\emptyset$,{D_4, D_5},{D_3, D_5},{D_3, D_4},{D_2, D_5}, {D_2, D_4},{D_2, D_3},{D_1, D_4},{D_1, D_3},{D_1, D_5},{D_1, D_2},{D_5},{D_4},{ D_3},{D_2},{D_1}} so $V(G\tau^C)$ = {{D_4, D_5},{D_3, D_5},{D_3, D_4},{D_2, D_5},{D_2, D_4},{D_2, D_3},{D_1, D_4},{D_1, D_3},{D_1, D_5},{ D_1, D_2},{D_5},{D_4},{ D_3},{D_2},{D_1}}.(As shown in Figure 4

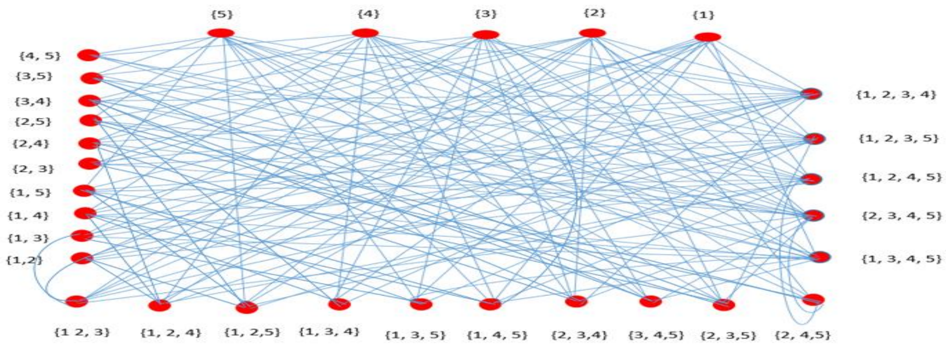


Fig. 4: complement topological graph where ∣X∣=5

Proposition 2.11: A complement of a discrete topological space graph is a simple graph.

Proof: By definition (2.1) let us assume that we have two vertices M and N in the topological space, which means that there are three possibilities: either M is part of N, or N is part of M, or there is no connection between them. Therefore, in the case of connection, there is only one edge between the two vertices, not repeated. Moreover, there is no vertex in the complement of the topological space that is connected to itself, i.e. it is not a loop. Therefore, the graph of the complement of the disjoint topological space is simple.

Proposition 2.12: A complement of a discrete topological space graph is an undirected graph.

Proof: From (definition 2.1), it becomes clear to us that the complement of a discrete topological space graph is an undirected graph.

Proposition 2.13: Suppose the cardinality of set X is denoted by $\mid X \mid = n$. In this case, the cardinality of $G\tau^C$ is determined to be $2^n - 2$.

Proof: Let τ^C complement the topology of X with cardinality 2 n. Consequently, the discrete topological graph $G\tau^C$ includes all elements from τ, excluding only the empty set $\emptyset$ and the set X, as per (Def.2.1).

Proposition 2.14: Let $\mid X \mid = n$ ($n \geq 3$), then $G\tau^C$ has no cut vertex.

Proof: Let us assume that v is a vertex in the graph $G\tau^{C}$, and let v be a vertex of one element. Referring to the definition of the complement of the discrete topological space, the vertex v is connected to vertices of two or more elements, of which it is a part, and so on for vertices that contain two elements and are also connected to individual vertices on the one hand. It is also connected to vertices that are part of it, and even vertex v_n contains n-1 elements and is connected to vertices that are part of it. After that, when vertex v is removed from the graph $G\tau$, a path can pass through another vertex, so the graph $G\tau^{C} - v$ It is connected, so v is not a cut vertex (see Example 2.8).

Proposition 2.15: Let $|X| = n$ ($n \geq 3$) and let $G\tau$ be a discrete topology on X then the independent number of $(G\tau^{C}) = ((n-1) + (n-2) + (n-3) + \cdots + n - (n-1))$

Proof: To calculate the independence number of the complement of a discrete topological graph, it must be noted that the graph's vertices are divided into several stages. The first stage includes vertices with single elements, the second includes vertices with two elements, and the third includes three elements until reaching stage (n -1) which includes (n-1) elements. According to the definition of the complement of the topological graph (Definition 2.1), the vertices of each stage are not adjacent to each other, which is what we want to achieve by creating independent groups whose elements are not adjacent, in this way, the mentioned stages are independent groups. The number of independence is the largest of the independent sets. See (Definition 2.2). Since we have the vertices of the second stage, it is the largest of the independent sums. For all stages, there is a discrete topology of x, so the formula $((n-1) + (n-2) + (n-3) + \cdots + n - (n-1))$ is used to calculate the number of vertices of the second stage, that is, the vertices with only two elements. By finding the number of these vertices, we guarantee obtaining the independence number for the complement discrete topological graph

Example 2.16: Let $X = \{D_1, D_2, D_3\}$, then $\tau = \{\emptyset, X, \{D_1\}, \{D_2\}, \{D_3\}, \{D_1, D_2\}, \{D_1, D_3\}, \{D_2, D_3\}\}$, and $\tau^{C} = \{X, \emptyset, \{D_2, D_3\}, \{D_1, D_3\}, \{D_1, D_2\}, \{D_3\}, \{D_2\}, \{D_1\}\}$ so $V(G\tau^{C}) = \{\{D_2, D_3\}, \{D_1, D_3\}, \{D_1, D_2\}, \{D_3\}, \{D_2\}, \{D_1\}\}$Since the number of non-adjacent vertices consisting of two elements is three vertices. According to the general formula mentioned in (Proposition 2.13), were $((n-1) + (n-2) + (n-3) + \cdots + n - (n-1))$ then the maximum independent set is $\{\{D_2, D_3\}, \{D_1, D_3\}, \{D_1, D_2\}\}$. The independent number of $G\tau^{C}$ is 3 (see Example 2.9).

Example 2.17 If $|X|$=4, then τ = $\{\emptyset, X, \{D_1\}, \{D_2\}, \{D_3\}, \{D_4\}, \{D_1, D_2\}, \{D_1, D_3\}, \{D_1, D_4\}, \{D_2, D_3\}, \{D_2, D_4\}, \{D_3, D_4\}\}$, and τ^C = $\{X, \emptyset, \{D_3, D_4\}, \{D_2, D_4\}, \{D_2, D_3\}, \{D_1, D_4\}, \{D_1, D_3\}, \{D_1, D_2\}, \{D_4\}, \{D_3\}, \{D_2\}, \{D_1\}\}$ so $V(G\tau^C)$ = $\{\{D_3, D_4\}, \{D_2, D_4\}, \{D_2, D_3\}, \{D_1, D_4\}, \{D_1, D_3\}, \{D_1, D_2\}, \{D_4\}, \{D_3\}, \{D_2\}, \{D_1\}\}$. Since the number of non-adjacent vertices consisting of two elements is sex vertices, and according to the general formula mentioned in (Proposition 2.13), were $((n-1) + (n-2) + (n-3) + \cdots + n - (n-1))$ then the independent number of $G\tau^C$ is {{3,4},{2,4},{2,3},{1,4},{1,3},{1,2}} =6. (See Proposition 2.7).

Example 2.18:
τ=$\{\emptyset, X, \{D_1\}, \{D_2\}, \{D_3\}, \{D_4\}, \{D_5\}, \{D_1, D_2\}, \{D_1, D_3\}, \{D_1, D_4\}, \{D_1, D_5\}, \{D_2, D_3\}, \{D_2, D_4\}, \{D_2, D_5\}, \{D_3, D_4\}, \{D_3, D_5\}, \{D_4, D_5\}\}$,and τ^C = $\{X, \emptyset, \{D_4, D_5\}, \{D_3, D_5\}, \{D_3, D_4\}, \{D_2, D_5\}, \{D_2, D_4\}, \{D_2, D_3\}, \{D_1, D_4\}, \{D_1, D_3\}, \{D_1, D_5\}, \{D_1, D_2\}, \{D_5\}, \{D_4\}, \{D_3\}, \{D_2\}, \{D_1\}\}$ so $V(G\tau^C)$ = $\{\{D_4, D_5\}, \{D_3, D_5\}, \{D_3, D_4\}, \{D_2, D_5\}, \{D_2, D_4\}, \{D_2, D_3\}, \{D_1, D_4\}, \{D_1, D_3\}, \{D_1, D_5\}, \{D_1, D_2\}, \{D_5\}, \{D_4\}, \{D_3\}, \{D_2\}, \{D_1\}\}$Since the number of non-adjacent vertices consisting of two elements is sex vertices, and according to the general formula mentioned in (Proposition 2.14), were $((n-1) + (n-2) + (n-3) + \cdots + n - (n-1))$

((5-1) + (5-2) + (5-3) + (5-4) = 4+3+2+1=10

then the maximum independent set is : $\{\{D_4, D_5\}, \{D_3, D_5\}, \{D_3, D_4\}, \{D_2, D_5\}, \{D_2, D_4\}, \{D_2, D_3\}, \{D_1, D_5\}, \{D_1, D_4\}, \{D_1, D_3\}, \{D_1, D_2\}\}$ and independent number of $G\tau^C$ is 10. (See Example 2.9).

Proposition 2.19: The chromatic number of $(G\tau^C)$ is $n-1$, where $n \geq 3$.

Proof: The chromatic number requires that every two adjacent vertices have a different color. It is important to know that the vertices in the graph take the form of one element, two elements, three elements, and even vertices in the form of $(n-1)$ elements, and for each category of these vertices (with one element, two elements, $(n-1)$ of the elements) have one color number because they are not adjacent according to the (Definition 2.1). We conclude, based on the enumeration of the head classes, that if we have $|X|= n$ $(n \geq 3)$ then the chromatic numbers for the graph of the topology complement is $(n-1)$.

Example 2.20: $X = \{D_1, D_2, D_3\}$, then By (Proposition 2.17), the chromatic numbers for the complement of discrete topological graph equal 2.

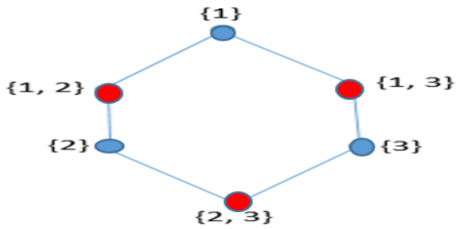
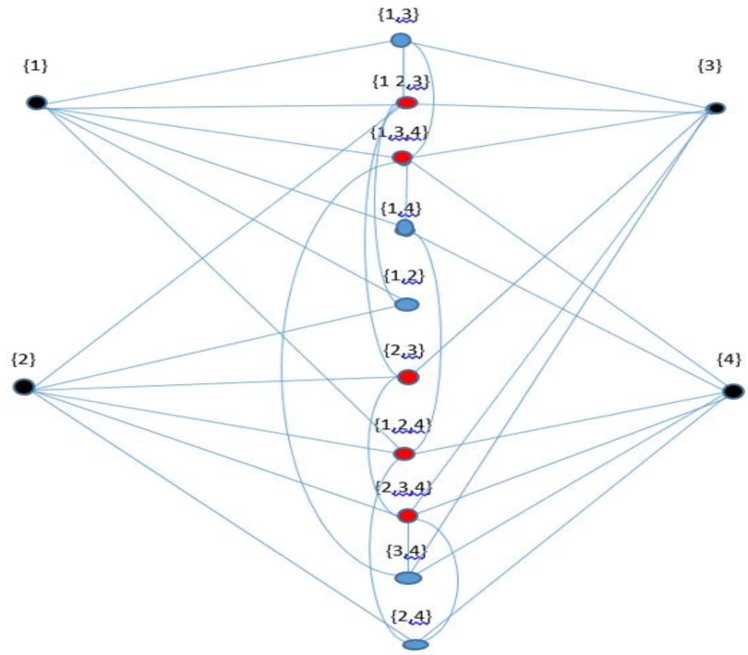


Fig. 5: The chromatic numbers graph $G\tau^{C}$ = 2.

Example 2.21: Let $X = \{D_1, D_2, D_3, D_4\}$, By (Proposition 2.17), the chromatic numbers for the complement of discrete topological graph equal 3.



Example 2.22: Let $X = \{D_1, D_2, D_3, D_4, D_5\}$,. Since by (Proposition 2.17), the color numbers for the complement of discrete topological graph equal 4.

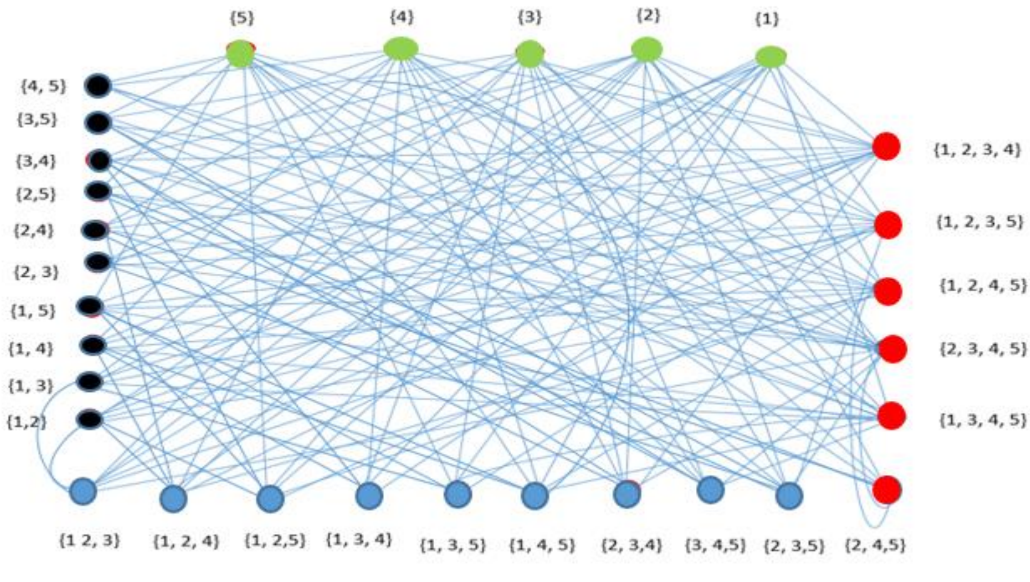
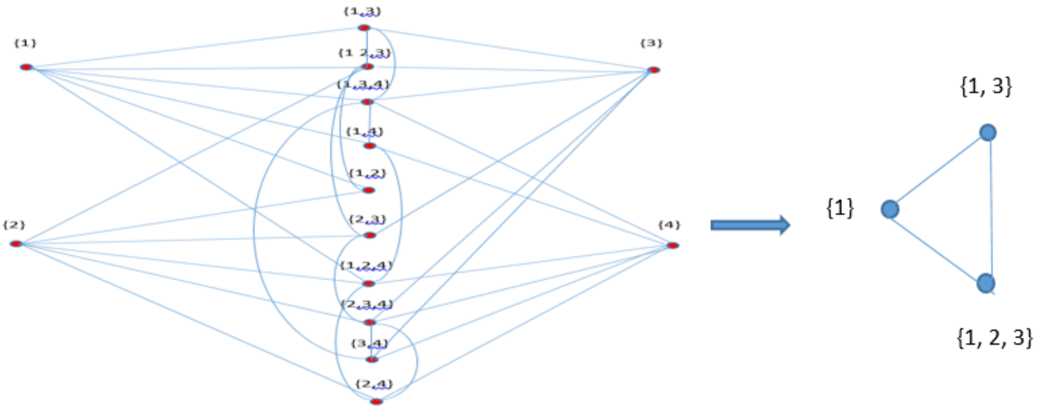


Fig. 7: The chromatic numbers graph $G\tau^{C}$ = 4.

Proposition 2.23: the girth number of $G\tau^{C}$ equals 3, where $n \geq 3$.

Proof: when $|X| = 3$, It is clear the Girth number equals 5(**See Figure 2**). moreover by (definition 2.1) According to the definition, every single vertex is the content of a two-element vertex, and every two-element vertex is the content of a three-element vertex. On the other hand, every single vertex is the content of a three-element vertex. So when $|X| = n$, (n > 3) then the least cycle is 3. (See Example 2.8).



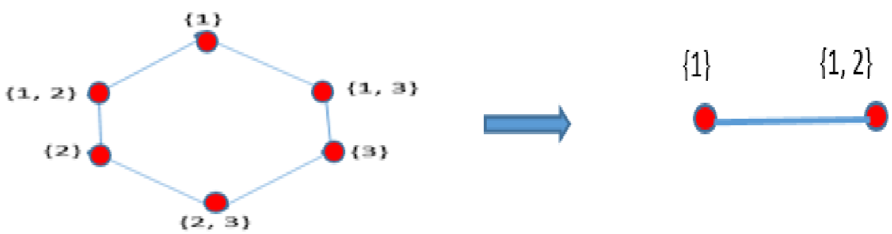
Graph $G\tau^{C}$ Girth of graph $G\tau^{C}$

Fig. 8: The Girth graph $G\tau^{C}$ is 3.

Proposition 2.24: then the Clique number of $G\tau^{C}$ equals $n-1$, where $n \geq 3$.

Proof: It is clear from the definition of the complement of a discrete topological graph that each vertex belongs to a vertex of two elements and also belongs to higher-order vertices and even vertices with $n-1$ elements. This partiality means the existence of adjacency between all these vertices up to the $n-1$ vertex because it excludes X from the set of vertices according to the definition of (Definition 2.5). Therefore the Clique number of $G\tau^{C}$ equals $n-1$.

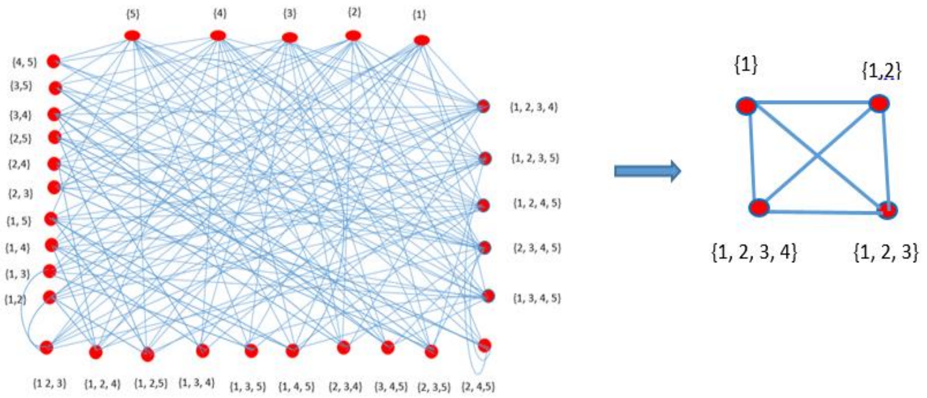
Example 2.25: If $|X|$=3, then the Clique number of $G\tau^{C}$ is 2.



Graph $G\,\tau^{C}$ Girth number of graph $G\tau^{C}$

Fig.9: The Girth graph $G\tau^{C}$ is 2.

Example 2.26: If∣X∣= 4, then the Clique number of graph $G\tau^{C}$ is 3. (See figure 7).
Example 2.27: If∣X∣= 5, then the Clique of $G\tau^{C}$ is 4**.**



Graph $G\tau^{C}$ Clique number of graph $G\tau^{C}$

Fig.10: The Clique number graph $G\ \tau^{C}$ is 4.

4. Conclusion

This paper aims to create a new approach: the complement of a topological graph derived from a discrete topology. The relationship between sets in a discrete topological space is the basis for how to write vertices and edges and find the relationship between them. Several properties of this graph were studied: a simple undirected, connected graph that does not have a cut vertex. Moreover, we calculated the independent numbers, chromatic number, girth number, and clique number for Complement discrete topological space, which is considered one of the important parameters in various studies such as networks, information systems, maps, etc.

5. Suggestions for fourth works:

We present our proposal to researchers interested in this field about the necessity of studying other topological spaces and describing them graphically, Such as studies on dominance, including strong, weak, complete, and independent are all concepts related to graph theory.

REFERENCES

1. Cvetkovic, Drago," Applications of graph spectra: An introduction to the literature," Application Graph *Spectra*, 2009. vol. 13, no. 21, pp. 7-31.
2. Malik, NR. *Graph theory with applications to engineering and computer science*. Courier Dover Publications, 2017. vol. 13, no.21, pp.1533-1543.

3. Kettani, Omar, "An algorithm for finding the Independence Number of a graph," arXiv preprint arXiv:0801.0590, 2008.
4. Idan, Mays K and Abdlhusein, Mohammed A, "Some Dominating Results of the Join and Corona Operations Between Discrete Topological Graphs," Int. J. Nonlinear Anal. Appl. accepted to appear, 2022. Vol.14, no.5, pp. 235—242.
5. Idan, Mays K and Abdlhusein, Mohammed A, "Different Types of Dominating Sets of the Discrete Topological Graph," Int. J. Nonlinear Anal. 2023. Appl., vol. 14, no. 1, pp. 101-108.
6. Jwair, Zainab n and Abdlhusein, Mohammed A, "The Neighborhood Topology Converted from the Undirected Graphs," Proceedings of IAM, 2022. vol. 11, no. 2, pp. 120-128.
7. Jwair, Zainab N and
8. Abdlhusein, Mohammed A, "Some Dominating Results of the Topological Graph," Int. J. Nonlinear Anal. Appl. accepted to appear, 2022. vol. 14, no. 2, pp. 133-140.
9. Jwair, Zainab Naeem and Abdlhusein, Mohammed Abdali. "Constructing new topological graph with several properties." Iraqi Journal of Science,2023. pp. 2991-2999.
10. Abdlhusein, Mohammed A and Al-Harere, Manal N, "New Parameter of Inverse Domination in Graphs," Indian J. Pure Appl. Math. ,2021., vol. 52, no. 1, pp. 281-288,
11. Liu, Yu and Lu, Jiaheng and Yang, Hua and Xiao, Xiaokui and Wei, Zhewei, "Towards maximum independent sets on massive graphs,". Association for Computing Machinery, 2015. vol. 8, no. 13, pp. 2122-2133
12. Obayes, Karrar Khudhair and Farawi, Yaqoob A and Obayes, Ghadeer Khudhair, "Using Graphs to Depict Relationships among Elements in Various Topological Spaces",Nanotechnology Perceptions,2024. pp. 800-808.
13. Poonkuzhali, S and Jayagopal, R. "Dominated coloring in certain networks." *Soft Computing*, Springer, 2024. pp. 1-9.
14. Wang, Tao and Yang, Xiaojing. "On odd colorings of sparse graphs." *Discrete Applied Mathematics* 345, 2024. vol.345, pp. 156-169.
15. Alaeiyan, Mehdi and Obayes, Karrar Khudhair and Alaeiyan, Mohammadhadi."Prediction nullity of graph using data mining".Results in Nonlinear Analysis,2023. vol. 6, no. 2, pp. 1-8.
16. Jwair, Zainab Naeem, and Mohammed Abdali Abdlhusein. "Constructing new topological graph with several properties." ,Iraqi Journal of Science , 2023.pp. 2991-2999.
17. Dharwadker, Ashay, "The Independent Set Algorithm," Institute of Mathematics, H-501 PalamVihar,2006.[Online].Available: https://www.dharwadker.org/independent_set/main.html. [Accessed 1 1 2023].
18. Maus, Yannic. "Distributed graph coloring made easy." ACM Transactions on Parallel Computing ,2023. vol. 10, no. 4, pp. 1-12.
19. Cho, Eun-Kyung, et al. "Odd coloring of sparse graphs and planar graphs." Discrete Mathematics ,2023 . vol. 346, no. 5, pp. 113305.
20. Kiss, György, Štefko Miklavič, and Tamás Szőnyi. "On girth-biregular graphs." Ars mathematica contemporanea ,2023. vol. 23, no. 4, pp. P4-01.
21. Garrett, Henry. Clique Number in Neutrosophic Graphs. Dr. Henry Garrett, 2023.