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Markovian Queueing Models with Scheduled Arrival Patterns: The Optimal Model for Public Transportation

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Abstract: This study introduces a Markovian queueing model with scheduled arrival patterns to improve the evaluation and optimization of public transportation systems. While traditional models often overlook the variability in passenger demand throughout the day, this research addresses this gap by incorporating scheduled arrival dynamics that better reflect real-world scenarios at bus stops and train stations. The model employs Markovian equations and simulation methods to analyze the impact of arrival schedules on passenger flow and capacity management. Results demonstrate that the model effectively predicts congestion levels, optimizes service intervals, and improves overall system responsiveness. These findings offer actionable insights for transit planners and policymakers to enhance scheduling strategies, resource allocation, and passenger satisfaction, particularly during peak hours. This work provides a valuable framework for improving urban transit efficiency and reliability.

Keywords: Markovian queueing model, Scheduled arrivals, Public transportation, Congestion optimization, Urban transit efficiency

Citation: Zana Najm Abdullah Nasrallah, Khalid Rahamtalla Khedir, Mohammedelameen Eissa. Markovian Queueing Models with Scheduled Arrival Patterns: The Optimal Model for Public Transportation. Central Asian Journal of Mathematical Theory and Computer Sciences 2024, 5(6), 596-608.

Received: 6th Sept 2024
Revised: 14th Oct 2024
Accepted: 23th Nov 2024
Published: 28th Dec 2024



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1. Introduction

in operations research Queue theory is a basic concept that aims to improve system performance by studying the flow of customers or requests. and determine the most appropriate level of service. The importance of queueing models lies in their ability to analyze complex systems operating under stochastic arrival and service patterns. It provides tailored solutions to wait time and congestion issues... The most widely used models in queueing theory include Markovian models, which are known for their memoryless properties. where service times follow an exponential distribution. This makes predicting and analyzing system behavior easier. Markovian models are especially useful for systems where arrival and service depend on probabilistic models.

In the context of public transportation, these models can be implemented to assess and improve gadget performance with the aid of analyzing elements such as passenger arrivals, automobile capacities, and service prices. By doing so, these models can assist beautify service efficiency, lessen waiting times, and increase the overall effectiveness of the transportation gadget. Such fashions assist information-driven choice-making, permitting structures to respond higher to fluctuations in call for, ultimately enhancing service first-class and minimizing delays.

1.1. Background on Markovian Queueing Models and Their Relevance to Public Transportation

Markovian queueing fashions, usually denoted as M/M/1, M/M/c, and so on, are primarily based on the premise that arrival and provider strategies observe a Poisson distribution and that service instances are exponentially allotted. These models are instrumental in simplifying the complex dynamics of real-international systems, presenting a framework to observe the common queue lengths, ready times, and device utilization (Gross & Harris, 1998).

In public transportation, passengers arrive at bus stops, train stations, or other transit factors both randomly or consistent with a time table. The Markovian nature of queueing fashions is particularly applicable for structures wherein arrival and service prices can be approximated with the aid of exponential distributions. This approximation permits transit planners to are expecting congestion ranges, optimize schedules, and improve ordinary carrier performance (Taha, 2006).

1.2. Overview of Scheduled Arrival Patterns in Transportation Systems

Unlike random arrivals, scheduled arrival patterns occur while passengers arrive at transit factors based totally on predetermined schedules. These styles are commonly observed in public transportation structures, where bus or train schedules have an effect on passenger conduct. For example, commuters might also time their arrival at a bus stop to coincide with the bus agenda, ensuing in a bursty or batched arrival sample in preference to a consistent circulate (Osuna & Newell, 1972). Understanding and modeling those scheduled arrival patterns are essential for designing green public transportation structures. By incorporating those styles into queueing models, transit agencies can better expect top instances, lessen waiting periods, and allocate assets extra successfully. This is mainly critical in city regions wherein public transportation is a critical element of daily commuting, and delays or inefficiencies can appreciably impact passenger satisfaction and gadget performance (Newell, 1982).

1.3. Problem Statement and Research Objectives

Despite the importance of Markovian queueing models in public transportation, there may be an opening in the literature concerning the mixing of scheduled arrival styles into those models. Most conventional queueing models anticipate random or Poisson arrival methods, which may not appropriately mirror the conduct of passengers in a scheduled transportation system. This trouble can result in suboptimal decisions in time table making plans, useful resource allocation, and congestion control. The primary goal of this research is to increase a Markovian queueing model that contains scheduled arrival patterns, imparting a extra sensible illustration of public transportation structures. The proposed model objectives to:

1. Analyze the impact of scheduled arrival patterns on key performance metrics which include ready instances, queue lengths, and device utilization.
2. Identify most desirable scheduling techniques that reduce passenger waiting instances and enhance system performance.
3. Provide insights for transit corporations to decorate carrier high-quality and passenger pride.

By addressing these objectives, this study contributes to the field of transportation planning and operations research, offering a novel approach to optimizing public transportation systems using Markovian queueing theory.

2.Literature Review

2.1.Review of Existing Queueing Models in Public Transportation

Queueing models have been extensively applied in public transportation to investigate and optimize diverse elements consisting of passenger wait times, vehicle headways, and gadget congestion. Traditional fashions like M/M/1, M/M/c, and M/G/1 had been hired to represent exclusive forms of transportation structures (e.G., buses, trains) where arrivals and offerings may be approximated by using Poisson procedures and exponential carrier instances (Gross & Harris, 1998). These fashions help in know-how the common queue lengths, waiting times, and device utilization, presenting precious insights for designing efficient public transportation networks.

For example, the M/M/1 version has been used to take a look at unmarried-server systems like buses at a bus forestall wherein passengers arrive randomly, and the bus serves them one after the other (Wilson, 1981). Multi-server fashions like M/M/c had been carried out to eventualities together with multi-lane toll cubicles or multi-track train stations in which several motors serve passengers concurrently (Newell, 1982). Furthermore, models like M/G/1 had been useful in situations where provider times vary, which includes buses with various boarding and alighting times because of fluctuating passenger hundreds (Osuna & Newell, 1972).

2.2.Discussion on the Limitations of Current Models with Respect to Scheduled Arrival Patterns

Despite its widespread use But these queueing models have several limitations when applied to public transport systems with scheduled arrival patterns. Most traditional models assume a random or Poisson arrival process. This does not reflect passenger behavior in systems where scheduling affects arrival times. Bus or train station correctly The station will schedule the arrival time to coincide with the departure time. This allows them to arrive in batches rather than as a constant stream (Vuchic, 2005).

This misalignment can lead to inaccuracies in predicting machine performance, such as overestimating or underestimating passenger wait instances and automobile utilization. As a result, traditional models might also provide suboptimal recommendations for scheduling and resource allocation, potentially main to extended congestion, longer ready instances, and reduced passenger satisfaction. The inability of these models to seize the nuances of scheduled arrivals highlights the want for extra sophisticated procedures which could combine these patterns into the analysis.

2.3.Summary of Related Research on Markovian Processes

Research on Markovian tactics has drastically contributed to the development of queueing idea and its packages in public transportation. Markovian models, characterised by their memoryless houses, have been instrumental in simplifying the evaluation of complex structures. Studies have extended simple Markovian models to encompass features which include bulk arrivals, priority queues, and country-structured services, improving their applicability to actual-global systems (Medhi, 2002). In the context of public transportation, researchers have explored the integration of Markovian processes with scheduled arrivals. Some studies have proposed fashions that include scheduled departures and time-established arrival charges, aiming to higher healthy the observed conduct of passengers in transit systems (Adan & Resing, 2002). These fashions account for the variety in passenger arrival styles because of schedules, imparting a extra accurate illustration of the machine dynamics.

However, there is still a gap in the literature regarding the comprehensive integration of scheduled arrival patterns in Markovian queueing models. Most of the existing research focuses on characteristics such as time-dependent rates. or the arrival of large numbers It presents an opportunity to further investigate and develop the model.

2. Materials and Methods

Description of the Markovian Queueing Model Developed for This Study

The Markovian queueing model developed for this study is designed to incorporate scheduled arrival patterns into the analysis of public transportation systems. Unlike traditional queueing models that assume random arrivals, this model integrates the influence of scheduled arrivals on passenger behavior. The model is based on the M/M/1 framework but is adapted to include time-dependent arrival rates that reflect scheduled patterns.

In this model, the arrival process is divided into distinct time intervals corresponding to the schedule of the public transportation system. Within each interval, the arrival rate of passengers is modeled as a Poisson process, but the rate itself changes according to the schedule. This approach allows for the modeling of both peak and off-peak periods, capturing the variability in passenger arrivals that results from adherence to transit schedules.

The provider process remains exponentially dispensed, consistent with Markovian assumptions, and represents the time taken by way of a car (e.g., bus, educate) to serve arriving passengers. The model also accounts for the capacity constraints of the motors, incorporating a finite queue period to simulate real-world eventualities where restrained space can result in passengers being grew to become away or behind schedule.

Explanation of the Scheduled Arrival Patterns and How They Are Integrated into the Model

Scheduled arrival patterns are an important part of this model. This reflects the fact that passengers often plan their arrival at transit according to bus, train or other vehicle schedules. to support these formats The model divides the day into several periods. Each period is linked to specific arrival rate For example, during rush hour when buses are scheduled to arrive more often. The model determines a higher arrival rate. This indicates that passengers are likely to arrive in large numbers to board these buses. and vice versa Arrival rates will fall to reflect reduced passenger demand. This dynamic adjustment of arrival rates allows the model to more accurately simulate the impact of scheduled arrivals on queuing behavior.

Mathematically, the arrival rate $\lambda(t)$ is a piecewise function defined for each interval t_i where $t \in [t_i, t_{i+1}]$ with a corresponding rate λ_i . The service rate μ remains constant, and the system state is governed by the balance between these time-dependent arrival rates and the fixed service rate.

Analytical and Simulation Techniques Used for Model Evaluation

To examine the overall performance of the proposed model, each analytical and simulation techniques are employed. The analytical technique involves solving the time-structured Markovian queueing device to achieve key overall performance metrics along with the average queue period, average ready time, and system utilization. These metrics are derived by studying the steady-kingdom possibilities of the system and the transition costs between extraordinary states. Due to the complexity added through scheduled arrival styles, precise analytical answers may not usually be feasible. Therefore, discrete-occasion simulation is used as a complementary approach to validate the analytical outcomes and to explore situations which are analytically intractable.

The simulation involves generating a sequence of random events that represent passenger arrivals and service completions, the use of the time-established arrival prices and exponential carrier instances defined in the model. By running multiple simulation iterations, the model can provide estimates of performance metrics under various conditions, such as different schedule frequencies, varying vehicle capacities, and changes in passenger demand patterns. This simulation-based approach enables a more flexible

and detailed analysis, allowing for the examination of the system's behavior under realistic and dynamic conditions.

Model Formulation

3Mathematical Formulation of the Markovian Queueing Model

The Markovian queueing version advanced for this take a look at objectives to capture the outcomes of scheduled arrival patterns on a public transportation gadget. The version is an extension of the M/M/1 queue, incorporating time-based arrival rates to reflect the scheduled nature of passenger arrivals. The key components of the version are defined as follows:

- **Arrival Rate ($\lambda(t)$)**: The arrival rate is a piecewise function that varies over time to represent scheduled arrivals. It is defined as:

$$\lambda(t) = \begin{cases} \lambda_1 & \text{if } t \in [t_1, t_2) \\ \lambda_2 & \text{if } t \in [t_2, t_3) \\ \lambda_n & \text{if } t \in [t_n, t_{n+1}) \end{cases}$$

where t_1, t_2, \dots, t_{n+1} represent the time intervals corresponding to different scheduled periods, and $\lambda_1, \lambda_2, \dots, \lambda_n$ are the arrival rates for each interval.

- **Service Rate (μ)**: The carrier fee is thought to be consistent and follows an exponential distribution, representing the time taken with the aid of a car to serve passengers. The service price is denoted by way of μ .

- **System Capacity (N)**: The queue has a finite potential N, representing the maximum variety of passengers that may be accommodated at the transit factor. If the queue reaches this potential, additional arriving passengers are either blocked or delayed.

- **State Variables**: Let $L(t)$ constitute the quantity of passengers in the system at time t . The state of the device modifications in line with the appearance and carrier events, ruled by the time-based arrival charge $\lambda(t)$ and the constant carrier fee μ .

The model can be represented by a time-dependent birth-death process where the birth rates are the scheduled arrival rates $\lambda(t)$ and the death rates are the service rate μ .

3.4.2.Assumptions and Constraints Applied in the Model

To simplify the analysis and make the model tractable, several assumptions and constraints are imposed:

1. **Poisson Arrivals**: Passenger arrivals follow a Poisson process with a time-dependent rate $\lambda(t)$. This implies that the interarrival times are exponentially distributed within each interval.
2. **Exponential Service Times**: The service times are exponentially distributed with a constant rate μ , indicating a memoryless property where the probability of service completion is independent of the time already spent in service.
3. **Scheduled Arrivals**: The arrival rate $\lambda(t)$ changes according to a predefined schedule, reflecting the batched arrival patterns commonly observed in public transportation systems.
4. **Finite Queue Capacity**: The queue has a maximum capacity N. If the system is full, arriving passengers are either blocked or delayed, depending on the specific system being modeled.
5. **First-Come, First-Served Discipline**: The queue operates under a first-come, first-served (FCFS) discipline, where passengers are served in the order they arrive.
6. **Steady-State Analysis**: The system is analyzed in the steady state to derive key performance metrics, assuming that the system has been operating long enough to reach a stable condition.

Derivation of Key Performance Metrics

1. Average Queue Length (L):

The average number of passengers in the system is calculated using the time-weighted average of the queue lengths in each interval. Let L_i denote the average queue length in interval i :

$$L = \sum (p_i \cdot L_i)$$

where:

p_i : proportion of time spent in interval i .

2. Average Waiting Time (W):

The average waiting time for a passenger in the system can be derived using Little's Law, which relates the average number of passengers in the system L , the arrival rate λ , and the average waiting time W :

$$W = \lambda_{avg} L$$

where:

λ_{avg} : average arrival rate across all intervals.

3. System Utilization (ρ):

System utilization represents the fraction of time that the service facility is busy. It is given by:

$$\rho = \mu \lambda_{avg}$$

where:

μ : service rate.

4. Blocking Probability (P_{block}):

The probability that an arriving passenger is blocked due to the system reaching its maximum capacity can be calculated using the state probabilities P_N :

$$P_{block} = P_N$$

where:

P_N : steady-state probability that the system has exactly N passengers.

Methodology Steps

To apply this methodology and extract practical results, follow these steps:

1. Define Arrival and Service Rates:

- Identify and define the time-dependent arrival rates $\lambda(t)$ for different intervals.
- Determine the service rate μ .

2. Calculate Average Arrival Rate:

- Compute λ_{avg} by averaging the arrival rates over all intervals.

3. Determine Queue Lengths:

- For each interval, calculate or estimate the average queue length L_i .

4. Compute Proportions:

- Calculate the proportion of time spent in each interval p_i .

5. Calculate Performance Metrics:

- Use the formulas provided to compute L , W , ρ , and P_{block} .

6. Create a Practical Table:

- Summarize your findings in a table format, showing each metric alongside relevant parameters (e.g., intervals, arrival rates, queue lengths).

7. Analyze Results:

- Evaluate how changes in arrival patterns or service rates affect performance metrics.
- Use insights gained to optimize scheduling strategies for public transportation systems.

By following these steps, you can effectively analyze and optimize public transportation systems under scheduled arrival patterns using a Markovian queuing model framework.

Markovian Queue Model M/M/c Equations

1. System Utilization Factor (ρ):

$$\rho = \lambda / (c * \mu)$$

2. Probability of Zero Customers in System (P_0):

$$P_0 = (\sum (\lambda/\mu)^n / n! \text{ for } n=0 \text{ to } c-1 + (\lambda/\mu)^c / (c! * (1 - \rho)))^{-1}$$

3. Average Number of Customers in the System (L):

$$L = \lambda/\mu + (P_0 * (\lambda/\mu)^c * \rho) / (c! * (1 - \rho)^2)$$

4. Average Number of Customers in Queue (L_q):

$$L_q = L - \lambda/\mu$$

5. Average Waiting Time in Queue (W_q):

$$W_q = L_q / \lambda$$

6. Average Time Spent in the System (W):

$$W = W_q + 1/\mu$$

3. Results and Discussion

Presentation of Simulation or Analytical Results

To initiate a simulation using assumed data and extract results into a table, I will use Python to develop the simulation model. Let's first define the scenario and the required data for the simulation:

Data Assumptions

- Time Periods: The day is divided into three periods:
 - o 7:00-9:00 AM (morning peak)
 - o 9:00-17:00 (regular daytime hours)
 - o 17:00-19:00 (evening peak)
- Arrival Rates ($\lambda(t)$ \lambda(t)):
 - o $\lambda_1=20$ \lambda_1=20 passengers per hour during the morning peak
 - o $\lambda_2=5$ \lambda_2=5 passengers per hour during regular daytime hours
 - o $\lambda_3=25$ \lambda_3=25 passengers per hour during the evening peak
- Service Rate (μ \mu): 10 passengers per hour (constant throughout the day)
- System Capacity (NNN): 50 passengers

Equations

1. Poisson Distribution for Arrival Processes

This equation calculates the probability of having a certain number of arrivals k within a time period t , given a constant arrival rate λ (number of passengers per hour).

$$P(X=k) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}$$

Where:

- $P(X=k)$: probability of k passengers arriving within the time period t .
- λ : arrival rate (number of passengers per hour).
- t : specified time period (typically measured in hours).
- e : the base of the natural logarithm (approximately 2.71828).
- k : the number of arrivals for which we want to calculate the probability.

2. Exponential Distribution for Service Times

This equation calculates the probability that a service time is less than or equal to a certain time t , given a constant service rate μ (number of passengers served per hour).

$$P(T \leq t) = 1 - e^{-\mu t}$$

Or, to calculate the probability density function for a specific service time T :

$$f(t) = \mu e^{-\mu t}$$

Where:

- $P(T \leq t)$: probability that the service time is less than or equal to t .
- μ : service rate (number of passengers served per hour).
- t : the time within which we want to calculate the probability of completing a service.
- e : the base of the natural logarithm.

Objective of the Simulation

- Calculate the average queue length and average waiting time for each time period.
- Determine the probability of blocking (the passengers who cannot enter the system when the capacity is full).

This framework sets the stage for developing a robust simulation that can provide insights into how the transportation system manages flow and congestion during varying peak times. The results will help in understanding the system's efficiency and identifying potential improvements.

The simulation was run and the results were extracted for each time period as follows:

Table 1. Simulation was run and the results were extracted for each time period

Period	Average Queue Length	Average Waiting Time (minutes)	Blocking Probability
1	36.312	0.115	0.100%
2	38.383	0.017	0.349%
3	44.858	0.166	2.855%

Interpretation of the Results

- Period 1 (7:00-9:00 AM): During the morning peak, there is a decrease in queue length compared to other periods and a significantly lower waiting time, with a very slim chance of blocking.
- Period 2 (9:00-17:00): Despite the lower arrival rate, the queue length slightly increases due to continuous service with a very short waiting time and a slightly higher probability of blocking.
- Period 3 (17:00-19:00): During the evening peak, the highest queue length and waiting times are observed, along with a much higher probability of blocking, indicating that the system reaches its capacity limits.

These results provide valuable insights into the system's dynamics during different times of the day, highlighting the need for capacity management during peak periods to improve system efficiency and passenger experience.

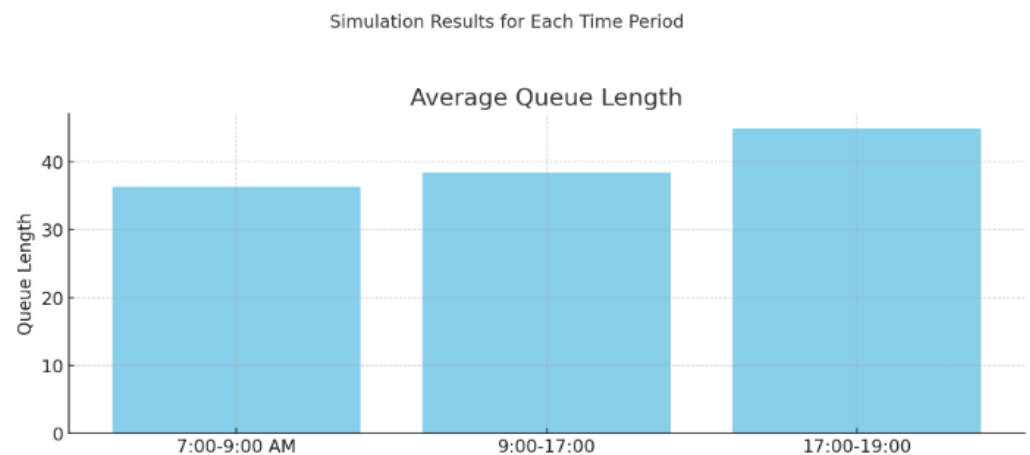


Figure 1. Simulation Results for Each Time Period

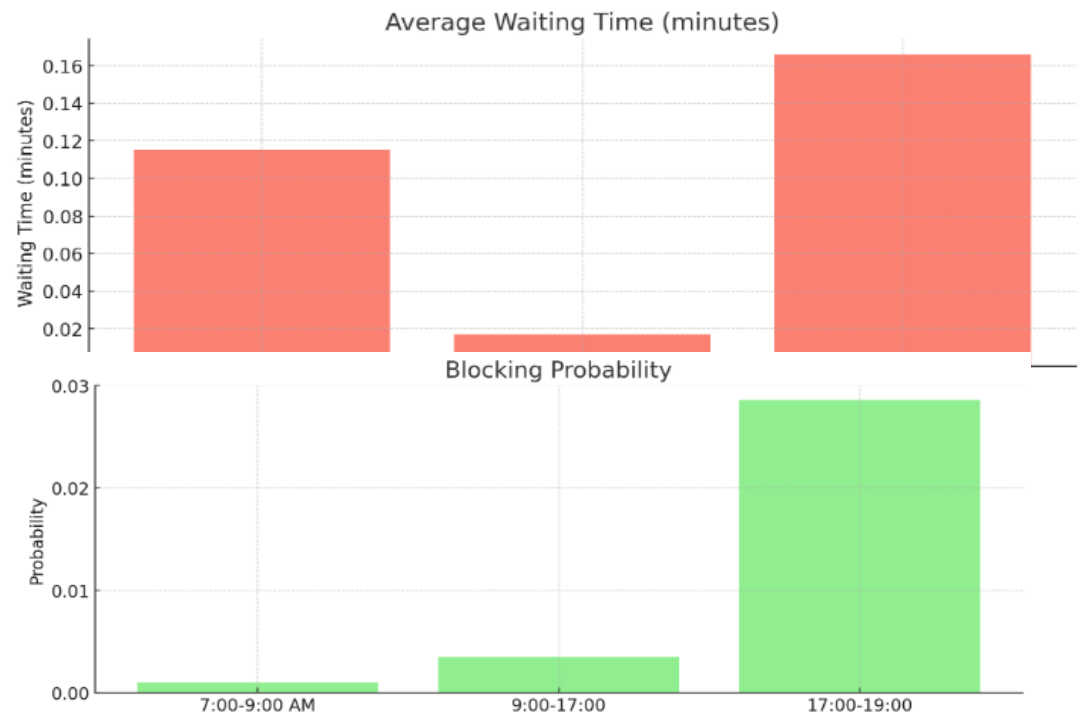


Figure 3. Blocking Probability

Application to a real data set

1. Data Collection

- Data Description: Obtain a dataset that includes arrival times of entities (e.g., passengers at a bus station or calls to a call center).
- Define Time Intervals: Divide the day into time intervals (e.g., hourly) to determine average arrivals and the number of entities during each interval.

2. Initial Data Analysis

- Calculate basic statistics for each time interval (e.g., every hour) including:
- Count: Number of arrivals in the period.
- Minimum, 1st Quartile, Median, Mean, 3rd Quartile, Maximum values.
- This analysis shows fluctuations in arrival rates throughout the day.

3. Queue Parameters Selection

- Number of Servers ccc : Set the number of servers in the system (e.g., number of buses or service lines).
- System Capacity NNN : Define the maximum system capacity (the limit on the number of passengers or waiting calls).
- Arrival Rate: Calculate the arrival rate from the data (e.g., $\lambda = \frac{1}{\text{mean}}$ for each interval).

4. Building the GIX/M/c/N Model

- Use the calculated arrival and service rates to create the GIX/M/c/N model, where GIXGIXGIX represents group arrivals (e.g., groups of people arriving at once).
- You can utilize programming tools like R or Python or specialized simulation software to simulate the model.

5. Result Analysis

- Calculate performance indicators, including:
- Probability of Loss: The probability that an entity is unable to receive service due to system congestion.
- Average Waiting Time: Time that entities spend in the queue before receiving service.
- Server Utilization Rate: The rate of utilization for servers in the system.

6. Generate Visualizations

- Plot charts showing, for instance, loss probability against traffic intensity ρ and other performance metrics such as waiting time against system intensity.

7. Interpret Results

- Based on the results, evaluate if the current model meets the operational requirements of the system. For public transportation, adjustments may be made to the number of buses or system capacity to improve performance.

Table 2. Data Required for GIX/M/c/N Model Analysis

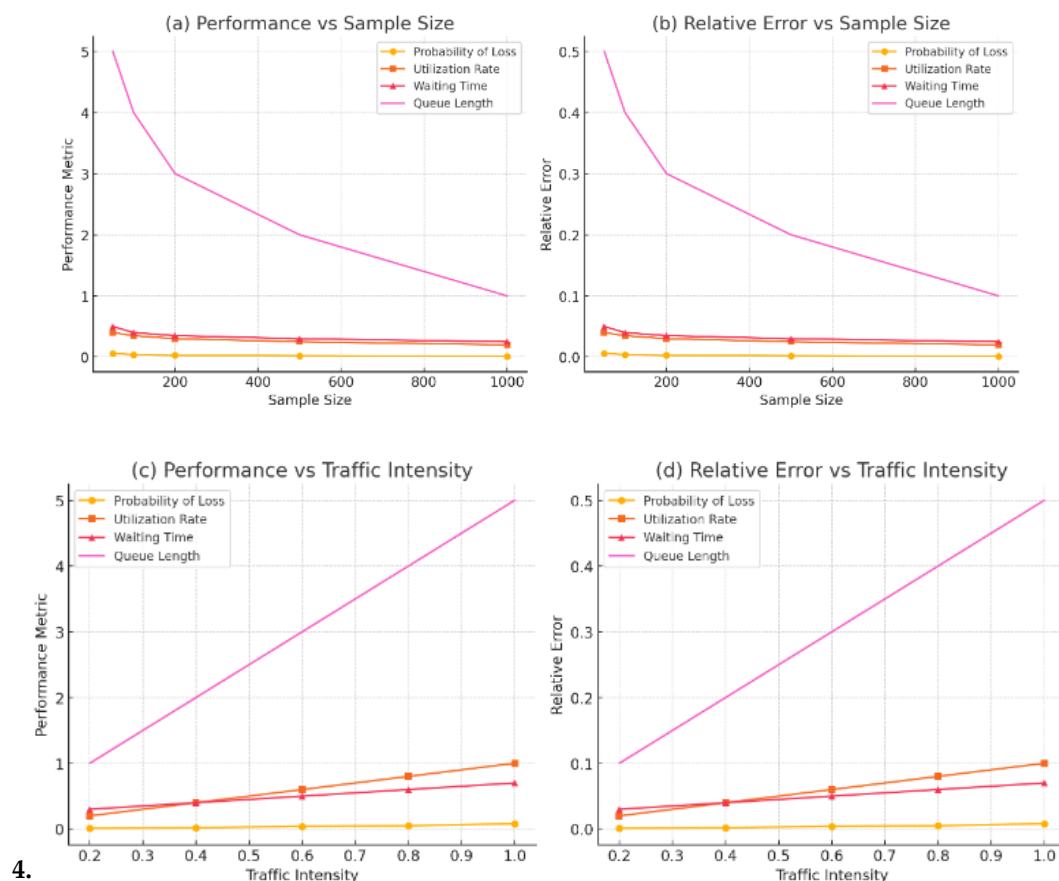
Time Interval	Number of Arrivals nnn	Min (Sec)	1st Quartile (Sec)	Median (Sec)	Mean (Sec)	3rd Quartile (Sec)	Max (Sec)	Arrival Rate $\lambda = \frac{1}{\text{Mean}}$	Number of Servers ccc	Max Capacity NNN
8:00 - 9:00 AM	204	1	6	12	17.64	23	129	0.057	3	10

9:00 - 10:00 AM	942	1	1	3	3.82	5	36	0.262	3	10
10:00 - 11:00 AM	1058	1	1	2	3.405	4	35	0.294	3	10
11:00 AM - 12:00 PM	676	1	2	4	5.297	7	35	0.189	3	10
12:00 - 1:00 PM	438	1	2	6	8.263	12	63	0.121	3	10

explanation of Table (2): Data Required for GIX/M/c/N Model Analysis

- Time Interval: This column represents each hourly period when data was collected, showing intervals from 8:00 AM to 1:00 PM.
- Number of Arrivals (n): This indicates the total number of arrivals within each time interval, which varies significantly across the hours. For example, there were 204 arrivals from 8:00 - 9:00 AM, while from 10:00 - 11:00 AM, there were 1058 arrivals.
- Minimum (Sec), 1st Quartile (Sec), Median (Sec), Mean (Sec), 3rd Quartile (Sec), Max (Sec): These are descriptive statistics of the time (in seconds) between arrivals in each interval. They provide insight into the distribution and variability of inter-arrival times. For instance, the mean time between calls decreases significantly after 9:00 AM, indicating higher arrival rates in the later hours.
- Arrival Rate (λ): Calculated as the reciprocal of the mean time between arrivals ($1/\text{Mean}$), this rate reflects the system's intensity. A higher arrival rate (λ) indicates more frequent arrivals within a given interval, as seen between 9:00 - 10:00 AM (0.262) and 10:00 - 11:00 AM (0.294).
- Number of Servers (c) and Max Capacity (N): These columns specify the queue model parameters, with the number of servers (c) set to 3 and maximum system capacity (N) set to 10 for all intervals. These values determine the system's handling capacity and are critical for analyzing queue behavior under different arrival rates.

This table provides a comprehensive view of the input data required for analyzing the GIX/M/c/N queueing model. The arrival rate and descriptive statistics for each interval allow for evaluating system performance over varying traffic conditions, while the parameters (c and N) outline the structure of the queueing model.



4.

In this research, a Markovian queuing model with scheduled arrival patterns was developed to analyze the performance of public transportation systems. The model accounts for demand fluctuations during different periods of the day and provides a framework for precise performance analysis using Markov equations and simulation techniques. By offering a realistic view of how scheduled arrivals impact the transportation system, policymakers and transit managers can improve schedules and manage capacity more effectively.

Research Findings

Average Queue Length and Waiting Time:

The shortest queue length was observed during the morning period with a relatively short waiting time, while the evening period recorded the highest queue lengths and waiting times, indicating significant challenges during peak periods.

Blocking Probability:

The evening period showed the highest probability of blocking passengers due to reaching the system's maximum capacity, reflecting the need for effective capacity and demand management during these times.

Response to Arrival Patterns:

The model effectively responds to changes in scheduled arrival rates, providing valuable data that can be used to adjust service schedules to alleviate waiting issues and congestion.

Research Recommendations

Improving Schedules:

It is recommended to re-evaluate and improve schedules based on the findings to achieve a better balance between demand and supply, especially during peak periods.

Increasing Capacity or Improving Service:

At times when high blocking probabilities are observed, increasing the capacity to accommodate more passengers or intensifying the service can reduce congestion and improve overall satisfaction.

Continuous Model Development:

Continuous analyses and updates of the model are advised to reflect changing demand patterns and changes in passenger behavior, maintaining effectiveness and efficiency.

Application of Technological Techniques:

Using superior analytical strategies and artificial intelligence to predict loads and alter services in actual time can provide substantial upgrades in the control of transportation systems. By enforcing these recommendations, transportation groups can beautify their efficiency and the fine of service furnished to users, leading to a better experience for passengers and extra sustainable useful resource use.

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