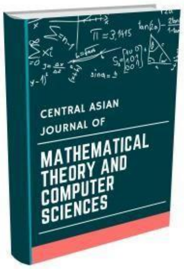

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Article

Exploring Numerical Methods: Solving Lane-Emden Type Equations with Padé Approximations

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Abstract: This research explores the effectiveness of using Padé approximations to enhance the accuracy of numerical solutions for Lane-Emden type differential equations. By applying the Adomian decomposition method to series solutions derived from previous studies, Padé techniques are integrated to obtain more precise approximate solutions. The supplied examples demonstrate that Padé approximations extensively outperform conventional strategies, yielding numerical results with smaller mistakes and nearer proximity to genuine solutions. Additionally, those approximations make a contribution to a higher information of the behavior of the studied structures by providing more stable and comprehensive answers. When in comparison to conventional answers, Padé approximations show off advanced performance throughout a number of situations, highlighting the importance of choosing the right numerical approach based on the nature of the hassle. This approach plays a crucial role in scientific and engineering fields that require high precision in modeling and analysis. Overall, the research emphasizes that Padé approximations represent an advanced and reliable option for addressing complex differential equations, opening new avenues for understanding mathematical and physical phenomena more effectively.

Keywords: Padé Approximation, Lane-Emden Equations, Numerical Methods, Mathematical Modeling, Adomian Decomposition Method

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1. Introduction

Padé approximation(Celik, E., Karaduman, E., & Bayram, M,2003), Turut, V., & Guzel, N,2012)) has been utilized in various fields for solving rational series. Studies have shown that Padé approximants outperform traditional series approximations, providing better numerical results compared to polynomial approximations. In this context, Wazwaz advanced a dependable algorithm based at the Adomian decomposition technique, which was carried out to Lane-Emden kind differential equations in papers (Wazwaz, A. M,2001). Successful collection answers for 2nd-order Lane-Emden type differential equations have been received. In this paper, Padé approximation is applied to those series answers derived by using Wazwaz in [5]. The algorithm, which relies on the Adomian decomposition method, is presented in (Yiğider, M., Tabatabaei, K., & Çelik, E. 2011), and section two of this paper will include brief information about this algorithm under the title "Analysis of the Method." This research aims to explore the effectiveness of Padé approximation in enhancing the accuracy of numerical solutions for Lane-Emden type differential equations, contributing to a better mathematical understanding of these equations and their applications in various fields.

Lane–Emden-type equations formulated as(Wazwaz, A. M,2002):

$$y" + \frac{2}{x}\, y' + f(y) = 0, \;\; 0 < x \leq 1$$

$$y(0) = A, \;\; y'(0) = B. \tag{1}$$

On the alternative hand, research have been carried out on some other class of singular initial value troubles of the form (Wazwaz, A. M,2002)

$$y' + \frac{2}{x} y' + f(x, y) = g(x), \;\; 0 < x \leq 1,$$

$$y(0) = A, \; y'(0)B. \tag{2}$$

In which A and B are constants, , f(x,y)is a non-stop actual-valued function, and g (x) is within the c language [0,1].

Equation (2) differs from the classical Lane–Emden type equations (1) in terms of the feature f (x,y) and the inhomogeneous time period g (x) (Wazwaz, A. M, 2002).

The info concerning Lane–Emden kind equations are sourced from Wazwaz (2002). For a greater comprehensive information of those equations, extra data may be located in Wazwaz (2001) and Wazwaz (2002).

2. Materials and Methods

The Adomian decomposition approach is considered an effective technique for solving differential equations, as it simplifies complicated equations by means of transforming them into operator shape. This technique specializes in the best-order derivative within the equation, making the solution procedure less difficult. According to Wazwaz (Wazwaz, A. M,2002), this method requires defining the differential operator L in a way that takes into account the two derivatives present in the problem, in order to overcome the singular behavior that may arise in certain cases.

The importance of this method will become in particular obtrusive in equations that show off unusual conduct or singular factors, as those factors can cause difficulties in locating solutions. By reformulating the equation in operator form, researchers can greater efficaciously address these challenges. Wazwaz (Wazwaz, A. M,2002) noted that rewriting equation (2) in the appropriate form can facilitate understanding of the solution behavior and enhance the accuracy of the results.

Furthermore, the use of the Adomian decomposition approach allows researchers to explore a huge range of possible answers, providing them with effective gear for studying dynamic structures. This method has been supported by way of previous studies, which have demonstrated how it is able to be efficiently implemented in diverse fields(Shawagfeh, N. T, 1993, Adomian, G,1986). Wazwaz (Wazwaz, A. M,2002) rewrote (2) inside the shape

$$Ly = -f(x, y) + g(x) \tag{3}$$

where the differential operator L is defined by

$$L = x^{-2} \frac{d}{dx}\left(x^2 \frac{d}{dx}\right). \tag{4}$$

Wazwaz [5] therefore considered the inverse operator L^{-1} as a two-fold integral operatör defined by

$$L^{-1}(.) = \int_0^x x^{-2} \int_0^x x^2 (.) dx dx. \tag{5}$$

Operating with L^{-1} on (3), Wazwaz [5] obtained

$$y(x) = A + Bx + L^{-1}g(x) - L^{-1}f(x, y). \tag{6}$$

The Adomian decomposition approach introduces the solution y(x) by an in countless collection of additives (Wazwaz, A. M,2002)

$$y(x) = \sum_{n=0}^{\infty} y_n(x), \tag{7}$$

and the nonlinear function $f(x, y)$ by an infinite series of polynomials

$$f(x,y) = \sum_{n=0}^{\infty} A_n$$

Where the components $y_n(x)$ of the solution y(x) will be determined recurrently, and An represents the Adomian polynomials that can be constructed for various classes of nonlinearity according to specific algorithms set by Adomian (Adomian, G,1992 , Adomian, G,1994)), and Adomian and Rach (Adomian, G., Rach, R., & Shawagfeh, N. T,1995)), and calculated by Wazwaz (Wazwaz, A. M,2000). For a nonlinear function F(u), the first few polynomials are provided, reflecting the properties of this function.

$$A_0 = F(u_0),$$
$$A_0 = u_1 F'(u_0),$$
$$A_2 = u_2 F'(u_0) + \frac{U_1^2}{2!} F''(u_0),$$
$$A_3 = u_2 F'(u_0) + u_1 u_2 F(u_0) + \frac{(u_1^3)}{(3!)} F''(u_0),$$
$$A_4 = u_4 F'(u_0) + \left(\frac{u_2^2}{2!} + u_1 u_2\right) F''(u_0) + \frac{(u_1^2)}{(2!)} F''(u_0) + \frac{1}{4!} u_1^4 F^{(iv)}(u_0),$$

Substituting (7) and (8) into (6) wazwaz [5] obtained

$$\sum_{n=0}^{\infty} y_n(x) = A + Bx + L^{-1} g(x) - L^{-1} \sum_{n=0}^{\infty} A_n \tag{10}$$

To determine the components yn(x) , Wawaz [5] used Adomian decomposition method that suggests the use of the recursive relatio

$$y_0(x) = A + BX + L^{-1} g(x),$$

$$y_{k+1}(x) + L^{-1}(A_k), \qquad K \geq 0, \tag{11}$$

which gives

$$y_0(x) = A + Bx + L^{-1} g(x),$$

$$y_1(x) = L^{-1}(A_0),$$

$$y_2(x) = -L^{-1}(A_1),$$

$$y_3(x) = -L^{-1}(A_2) \tag{12}$$

Wazwaz (Wazwaz, A. M,2002) combined the scheme (12) with (9) that will enable us to determine the components $y_n(x)$ recursively, and hence the series solution of y(x) defined by (7) follows immediately. For numerical purposes, the n -term approximant

$$\backslash\emptyset_n = \sum_{k=0}^{n-1} y_k, \tag{13}$$

can be used to approximate the solution [5].

Padé approximation

Consider a formal power series

$$f(x) = c_0 + c_1x + c_2x^2 + \ \ldots. \tag{14}$$

with ($c_0 \neq 0$) (Cuyt, A., & Wuytack, L, 1987). In this paper ∂p is written for the exact degre of a polynomial p and ω p for the order of a power series p [12]. (i.e the degree of the first nonzero term).

The Padé approximation problem of order (m, n) or [m,n] for f consists in finding polynomials

$$p(x) = \sum_{i=0}^{m} a_i x^i \,, q(x) = \sum_{i=0}^{n} b_1 x^i \tag{15}$$

Such that in the power series (fq -p) (x) - the coefficients of x^i for 0, ... m+n disappar, i.e (Cuyt, A., & Wuytack, L, 1987).

$$\partial(p) \leq m$$

$$\partial(q) \leq n$$

$$\omega(fq - p) \geq m + n + 1 \tag{16}$$

Condition (16) is equivalent with the following two linear systems of equations

$$\begin{cases} c_0b_0 = 0 \\ c_1b_0 + c_0b_1 + a_1 \\ \vdots \\ c_mb_0 + c_{m-1}b_1 + \ \ldots + c_{m-n}b_n = a_m \end{cases} \tag{17}$$

$$\begin{cases} c_{m+1}b_0 + c_mb_1 + \ \ldots + c_{m-n+1}b_n = a_m \\ \vdots \\ c_{m+n}b_0 + c_{m-n+1}b_1 + \ \ldots + c_mb_n = 0 \end{cases} \tag{18}$$

with c_1= 0 for i < 0 [12]. For n = 0 the systems of equations (18) is empty. In this case,

$a_1 = c_1$ (i =0, ..., m) and b_0 =1 satisfy (16), in the other words the partial sums of (14) solve the Padé approximation problem of order (m,0) .

In general a solution for the coefficients a_1 is known after substitution of a solution for the bi in the left hand side of (17). So the crucial point is to solve the homogeneous system of n equations (18) in the n +1 unknowns b_1 . This system has at least one nontrivial solution because one of the unknowns can be chosen freely (Cuyt, A., & Wuytack, L, 1987).

In short, by solving the equations (17) and (18) the coefficients a_1 and b_1 are found. Then the Padé equations (15) are found. After finding these polynomials we get The Padé approximation of order (m, n) or [m, n] for f.

3. Results and Discussion

In this section Padé series solutions of differential equations of Lane-Emden type shall be illustrated by two examples.

Example 4.1.

Consider the nonlinear singular initial value problem[5]:

$$y" + \frac{2}{x}\, y' + 4(2e^{y} + e^{y/2}) = 0,$$

y(0) = 0, y'(0) = 0 (19)

Wazwaz solved equation (19) by using the method that we mentioned in section 2 and

$$y(x) = 2\left(x^2 - \tfrac{1}{2}x^4 + \tfrac{1}{3}x^6 - \tfrac{1}{4}x^8 + \tfrac{1}{5}x^{10} - \tfrac{1}{6}x^{12} + \ \ldots.\right) \quad (20)$$

Series solution has been obtained in(Wazwaz, A. M,2002). For more information about the solution of equation (19) by Wazwaz can be seen in (Wazwaz, A. M,2002). Exact solution for equation (20) is given as f$f(x) = -2\, In\, (1 + x^2)$ in (Wazwaz, A. M,2002).

If the Padé approximation is applied to solution (20) then Padé series of different orders can be obtained. By applying Padé approximation the following Padé approximation of order (Wazwaz, A. M, 2001), and (Turut, V., & Guzel, N,2012) for equation (19) are obtained:

$$[4,4]_{y(x)} = \frac{-x^4-2x^2}{\frac{1}{6}x^4+x^2+1} \quad (21)$$

$$[6,4]_{y(x)} = \frac{\frac{1}{15}x^6-\frac{1}{15}x^2-2x^2}{\frac{3}{10}x^4+\frac{6}{5}x^2+1} \quad (22)$$

$$[8,2]_{y(x)} = \frac{-\frac{1}{30}x^8+\frac{2}{15}x^6-\frac{3}{5}x^4-2x^2}{\frac{4}{5}x^2+1} \quad (23)$$

To obtain the Padé series (21), (22), and (23) the solution tecnique that mentioned in section 3 for the linear systems of equations (17) and (18) is can be applied. If the numerical results are compared for example 1, the following tables and figures are obtained (Table 1 and figure 1, figure 2, figue 3);

Example 4.2.

Consider the nonlinear singular initial value problem[5]:

$$y" + \frac{6}{4}\, y' + 14y = -4y\, Iny,$$

y(0) = 1, y' (0) = 0. (24)

Wazwaz solved equation (23) by using the method that we mentioned in section 2 and

$$y(x) = 1 - x^2 + \tfrac{1}{2!}x^4 - \tfrac{1}{3!}x^6 + \tfrac{1}{4!}x^8 - \tfrac{1}{5!}x^{10} + \ \ldots\ldots.. \quad (25)$$

Series solution has been obtained in [5]. More information about the solution of equation (24) by Wazwaz can be seen in [5]. Exact solution for equation (24) is given as $f(x) = e^{-x^2}$ in [5].

If the Padé approximation is applied to solution (25) then Padé series of different orders can be obtained. By applying Padé approximation the following Padé approximation of order [4,4] , [6,4] and [8,2] for equation (25) are obtained:

$$[4,4]_{y(x)} = \frac{\frac{1}{2}x^4-\frac{1}{2}x^2+1}{\frac{1}{12}x^4+\frac{1}{2}x^2+1} \quad (26)$$

$$[6,4]_{y(x)} = \frac{-\frac{3}{5}x^2+\frac{3}{20}x^4-\frac{1}{60}x^6+1}{\frac{1}{20}x^4+\frac{2}{5}x^2+1} \quad (27)$$

$$[8,2]_{y(x)} = \frac{1-\frac{4}{5}x^2+\frac{3}{10}x^4-\frac{1}{15}x^6+\frac{1}{120}x^8}{\frac{1}{5}x^2+1} \quad (28)$$

To obtain the Padé series (26), (27), and (28) the solution tecnique that mentioned in section 3 for the linear systems of equations (17) and (18) is can be applied. If the numerical results are compared for example 2, the following tables and figures are obtained (Table 2 and figure 3, figure 4 ,figure 5).

4. Conclusion

The numerical results presented in the tables (Table 1, Table 2) and figures (Figure 1, Figure 2, Figure 3, Figure 4, Figure 5, Figure 6) demonstrate that Padé approximations significantly outperform series approximations. Padé approximations yield accurate results with smaller error bounds, making them a reliable choice for obtaining better numerical values.

Padé approximations are a powerful tool in the analysis of mathematical equations, characterized by their ability to provide precise solutions even in cases where series approximations may be less effective. By evaluating the consequences received from Padé approximations with those derived from collection approximations, it will become obvious that Padé offers better accuracy and higher performance throughout numerous situations.

Furthermore, the use of Padé approximations can contribute to a deeper information of the behavior of the studied structures, as this method lets in for a extra comprehensive exploration of solutions. The results obtained via these approximations beautify the reliability of the mathematical fashions hired, permitting researchers to make informed selections based totally on numerical information.

In light of the above, it is able to be concluded that Padé approximations constitute a favored alternative in lots of medical and engineering packages, presenting more accuracy and reliability as compared to series approximations. Therefore, adopting this method can cause stepped forward consequences and a deeper knowledge of the phenomena below investigation.

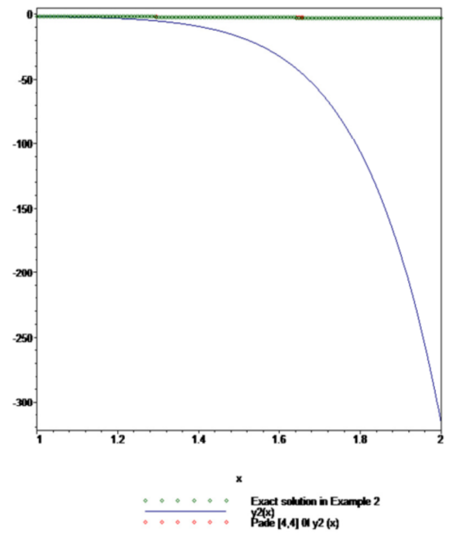

Figure 1. Presents the exact solution of equation 19 in Example 1, along with the series solution y(x) from the same example. It additionally includes the [4,4] Padé approximation of the y(x) series answer, taking into account a contrast of the effectiveness of the special solutions

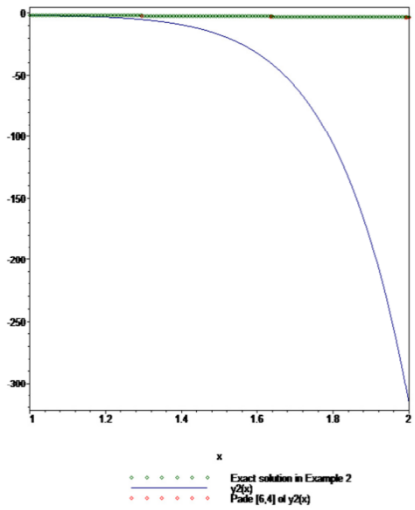

Figure 2. Illustrates the exact solution of equation 19 from Example 1, alongside the series solution. It additionally features the [6/4] Padé approximation of the y(x) collection answer, imparting a foundation for comparing the accuracy and performance of the numerous solution strategies employed.

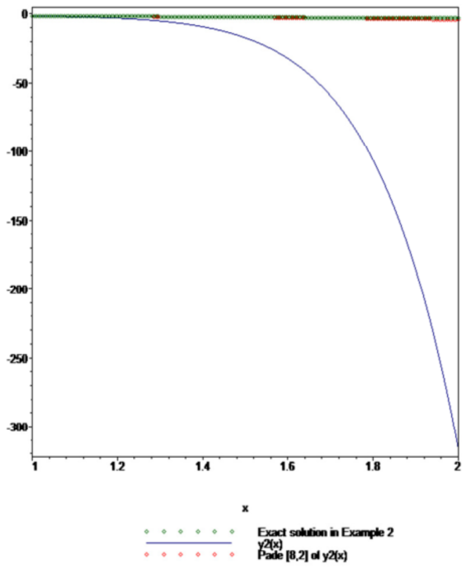

Figure 3. Exact solution of equation 19 in Example 1, series solution in Example [8/2] Padé approximation of y(x) series solution ($[8/2]_{y(x)}$).

Table 1. Numerical and absolute error values for Example 1.

x	Exact solution $f(x)=-2\ln(1+x^2)$	y(x)	$[4/4]_{y(x)}$	$[6/4]_{y(x)}$	$[8/2]_{y(x)}$	$\lvert-2\ln(1+x)-y(x)\rvert$	$\lvert-2\mathrm{In}(1+x^2)-(4/4)\rvert_{y(x)}$	$\lvert-2\mathrm{In}(1+x^2)-[6/4-]_{y(x)}\rvert$	$\lvert-2(1+x^2)-[8/2]_{y(x)}\rvert$
1.0	-1.386294361	-1.566666667	-1.384615385	-1.386666667	-1.388888889	0.180372306	0.001678976	0.000372306	0.002594528
1.1	-1.585985031	-2.102643246	-1.582752086	-1.586813940	-1.592329689	0.516658215	0.003232945	0.000828909	0.006344658

1.2	-1.783996079	-3.123842089	-1.778288340	-1.785660522	-1.798027896	1.339846010	0.005707739	0.001664443	0.014031817
1.3	-1.979082387	-5.177458734	-1.969699043	-1.982152956	-2.007651236	3.198376347	0.009383344	0.003070569	0.028568849
1.4	-2.170378536	-9.289331587	-2.155840308	-2.175658248	-2.224669607	7.118953051	0.014538228	0.005279712	0.054291071
1.5	-2.357309992	-17.28281250	-2.335877863	-2.365868263	-2.454659598	14.92550251	0.021432129	0.008558271	0.097349606
1.6	-2.539521090	-32.25683931	-2.509228476	-2.552718922	-2.705660640	29.71731822	0.030292614	0.013197832	0.166139550
	-2.716818316	-59.28058147	-2.675512194	-2.736324019	-2.988577883	56.56376315	0.041306122	0.019505703	0.271759567
1.8	-2.889126538	-106.3761022	-2.834513156	-2.916922342	-3.317627224	103.4869757	0.054613382	0.027795804	0.428500686
1.9	-3.056455714	-185.8766558	-2.986147188	-3.094836308	-3.710818814	182.8202001	0.070338526	0.038380594	0.654363100
2.0	-3.218875824	-316.2666667	-3.130434783	-3.270440252	-4.190476190	313.0477909	0.088441041	0.051564428	0.971600366

Table 1 affords the numerical values and absolute mistakes associated with fixing equation 19 in Example 1. It includes the exact answer for the feature f(x) = -2ln(1 x^2), along with the collection answer y(x) and Padé approximations of levels [4/4], [6/4], and [8/2]. By reading the records, we can draw several conclusions approximately the accuracy of every of those answers.

First, the precise solution f(x) serves as a reliable reference, providing steady effects across more than a few x values. For instance, at x = 1.0, the exact cost is -1.386294361, at the same time as the numerous approximations yield close results, indicating that the approximations can successfully simulate the general behavior of the function. However, as the fee of x increases, the gap between the approximate solutions and the precise answer widens, suggesting that the accuracy of the approximations can be greater affected at higher x values.

When analyzing the Padé approximations, we find that the [4/4] approximation provides consequences very close to the exact solution, with a fee of -1.384615385 at x = 1.Zero. However, other approximations, inclusive of [6/4] and [8/2], show greater variability in their accuracy. For instance, at x = 1.5, the [6/4] approximation yields -2.365868263, that is further from the precise fee of -2.357309992, indicating that this approximation can be much less correct in this situation.

Analyzing the absolute mistakes, we take a look at that the absolute variations among the exact answer and the approximations increase with growing x values. For example, at x = 2.0, the distinction among the exact answer and the [8/2] approximation reaches 0.971600366, indicating that the approximations might also end up much less correct in positive stages. This displays the demanding situations confronted by means of approximations in appropriately simulating the right conduct of mathematical functions at unique values.

Furthermore, it's far major that the absolute errors increase significantly with better x values. At x = 1.Nine, the difference between the precise answer and the [6/4] approximation is zero.038380594, suggesting that the approximations may additionally end up less accurate in positive ranges. This shows that the use of Padé approximations requires warning, particularly when dealing with larger x values.

Overall, the results highlight the importance of selecting the best approximation based at the range of values being studied. While Padé approximations provide exact effects, the exact solution remains the maximum reliable. Therefore, it's miles essential to consider the accuracy of the approximations when the use of them in sensible programs, mainly in cases that require high precision.

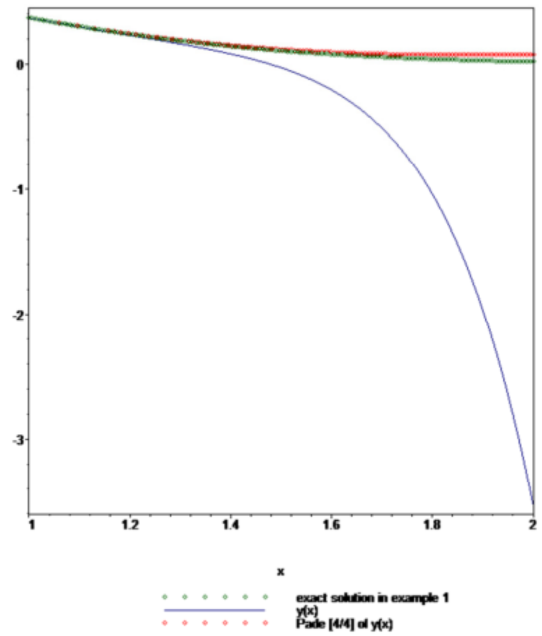

Figure 4. Shows the exact solution of equation 24 in Example 2 alongside the series solution y(x). It also presents the [4/4] Padé approximation of the y(x) series, enabling comparison of accuracy and effectiveness between the exact solution, the series, and the Padé approximation.

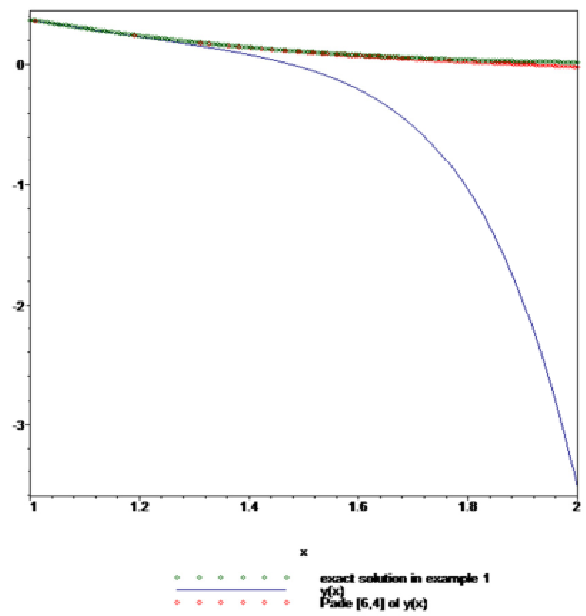

Figure 5. Exact solution of equation 24 in Example 2, y(x) series solution in Example 2 [6/4] Padé approximation of y(x) series solution ($[6/4]_{y(x)}$).

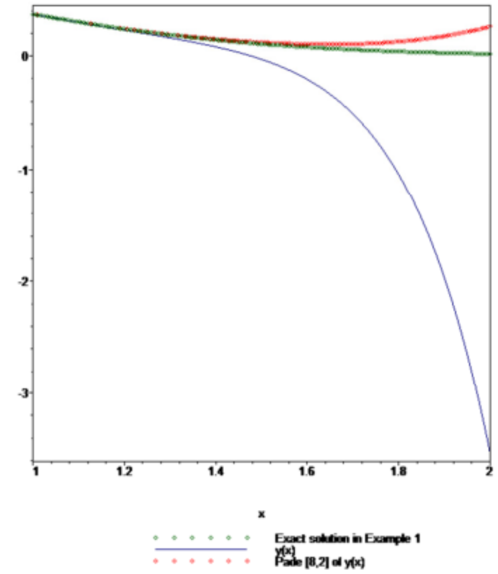

Figure 6. displays the exact solution of equation 24 in Example 2 along with the series solution y(x). Additionally, it includes the [6/4] Padé approximation of the y(x) series, allowing for a comparison of the approximation's accuracy relative to the exact solution and the original series.

Table 2. Numerical and absolute error values for Example 2.

x	Exact solution	y(x)	[4/4] $_{y(x)}$	[6/4] $_{y(x)}$	[8/2]$_{y(x)}$	$\lvert e^{-z^2}y(x)\rvert$	$\lvert e^{-z^2}[4/4]_{y(z)}\rvert$	$\lvert e^{-z^2}[6/4]_{y(z)}\rvert$	$\lvert e^{-z^2}[8/2]_{y(z)}\rvert$
1.0	0.3678794412	0.3666666667	0.3684210526	0.3678160920	0.368055556	0.0012127745	0.0005416114	0.0000633492	0.0001761144
1.1	0.2981972794	0.2944915132	0.2993664381	0.2980269029	0.2987030382	0.0037057662	0.0011691587	0.0001703765	0.0005057588
1.2	0.2369277587	0.2266972365	0.2392223161	0.2365174319	0.2382346335	0.0102305222	0.0022945574	0.0004103268	0.0013068748
1.3	0.1845195240	0.1585875572	0.1886734331	0.1836195652	0.1876085900	0.0259319668	0.0041539091	0.0008999588	0.0030890660
1.4	0.1408584209	0.00797438939	0.1478754855	0.1390367461	0.1476245364	0.0611145270	0.0070170646	0.0018216748	0.0067661155
1.5	0.1053992246	-0.0298583982	0.1165644172	0.1019593614	0.1192753233	0.1352576228	0.0111651926	0.0034398632	0.0138760987
1.6	0.07730474044	-0.2060926500	0.09416871108	0.07119154532	0.1041751814	0.2833973904	0.01686397064	0.00611319512	0.02687044096
1.7	0.05557621261	-0.510307484	0.07991329748	0.04527586141	0.1050518714	0.5658836966	0.02433708487	0.01030035120	0.04947565879
1.8	0.3916389510	-1.043643114	0.07290832093	0.02260627889	0.1262915342	1.082807009	0.03374442583	0.01655761621	0.08712763910
1.9	0.02705184687	0.07221992578	0.07221992578	0.001523767826	0.1745266278	1.994719296	0.04516807891	0.02552807904	0.1474747809
2.0	0.01831563889	-3.533333333	0.07692307692	0.01960784314	0.2592592593	3.551648972	0.05860743803	0.03792348203	0.2409436204

Table 2 presents the numerical values and absolute error values for Example 2, where the exact solution is given by the function($f(x) = e^{-z^2}$). The table includes the series solution (y(x)) and various Padé approximations: [4/4], [6/4], and [8/2]. This comprehensive dataset allows for a detailed analysis of the accuracy of each approximation compared to the exact solution.

The actual answer(f(x)= 〖e^(-z)〗 ^2) is a well-known feature that describes the exponential decay of a squared variable. As (z) increases, the cost of (f(x)) decreases swiftly, which is obvious in the values presented within the table. For instance, at (z = 1.0), the exact value is approximately 0.3678794412, and it decreases to 0.01831563889 at (z =

2.Zero). This behavior is feature of exponential features, wherein the output diminishes appreciably because the input increases.

The series answer (y(x)) offers an approximation of the precise solution. At (z = 1.0), the collection solution yields zero.3666666667, that's pretty close to the exact value, resulting in a small absolute mistakes of 0.0012127745. This suggests that the series answer is effective for small values of (z). However, as (z) increases, the accuracy of the collection answer begins to decline.

For example, at (z = 1.Four), the collection solution drops to zero.00797438939, that is substantially decrease than the precise value of zero.1408584209, leading to a bigger absolute errors of zero.0611145270. This trend keeps as (z) will increase, with the series answer diverging greater from the exact answer, in particular obtrusive at (z = 1.6) and beyond, wherein the collection solution turns into bad.

The Padé approximations provide a extraordinary method to approximating the exact answer. The [4/4] approximation yields values which might be generally toward the exact solution than the collection solution, particularly for decrease values of (z). For instance, at (z = 1.0), the [4/4] approximation gives 0.3684210526, ensuing in a minimum absolute error of 0.0005416114. This demonstrates the effectiveness of the [4/4] Padé approximation in shooting the behavior of the precise answer.

As we pass to the [6/4] and [8/2] approximations, we have a look at varying degrees of accuracy. The [6/4] approximation at (z = 1.0) yields zero.3684210526, with a totally small mistakes of zero.0000633492, indicating high accuracy. However, as (z) will increase, the overall performance of those approximations starts offevolved to differ. For example, at (z = 1.5), the [6/4] approximation offers zero.1165644172, that's in the direction of the precise cost than the series solution however nonetheless results in a major mistakes of 0.1352576228.

The [8/2] approximation shows a comparable trend, presenting values that are generally closer to the exact solution than the collection solution, but with growing mistakes as (z) rises. At (z = 2.Zero), the [8/2] approximation yields zero.2592592593, leading to a good sized absolute blunders of 0.2409436204, indicating that at the same time as Padé approximations can be powerful, additionally they have limitations at better values of (z).

The absolute errors for each method reveal critical insights into their overall performance. For decrease values of (z), both the series answer and Padé approximations maintain fantastically small mistakes. However, as (z) increases, the errors for the series answer develop notably, particularly after (z = 1.5). The Padé approximations, whilst initially extra correct, additionally start to diverge from the exact answer, particularly the [8/2] approximation, which indicates the most important mistakes at better (z) values.

At (z = 1.8), absolutely the errors for the collection answer reaches 1.082807009, indicating a sizeable deviation from the exact solution. This highlights the demanding situations of the usage of series expansions for approximating capabilities that showcase fast adjustments, along with exponential decay.

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