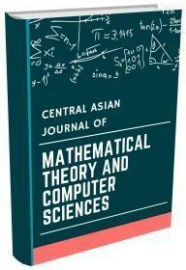

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Article

Robust Variable Selection for Quantile Regression: Application to Daily Demand Forecasting Orders Data

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Abstract: This paper proposes two robust variable selection methods within the quantile regression framework: Robust Elastic Net Quantile Regression (REN-QR) and Robust MCP Quantile Regression (R-MCP-QR). These approaches integrate adaptive penalization with GM-type weighting schemes to improve estimation accuracy and feature selection under high-dimensional and contaminated conditions. Through extensive simulation studies, the proposed methods demonstrate superior performance in terms of mean squared error (MSE), true positive rate (TPR), and false positive rate (FPR) compared to classical penalized quantile regression techniques. Furthermore, the application to a real-world dataset on daily demand forecasting orders confirms their effectiveness in capturing relevant predictors while maintaining robustness against outliers. The results highlight the utility of robust penalized quantile regression for accurate and interpretable modeling in complex data environments.

Keywords: Quantile Regression, Robust Variable Selection, Elastic Net, MCP Penalty, Demand Forecasting

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1. Introduction

Quantile regression (QR), introduced by Koenker and Bassett (1978), has become a widely used statistical framework for modeling the conditional quantiles of a response variable. Unlike classical least squares, which focuses on the conditional mean, QR enables the analysis of the entire conditional distribution, making it particularly useful in the presence of heteroscedasticity, non-normality, and outliers (Koenker, 2005). This feature is crucial for forecasting and decision-making applications where the distributional behavior of the response is not symmetric.

In high-dimensional settings, variable selection plays a central role in improving interpretability and reducing prediction error. Penalized regression techniques such as the LASSO (Tibshirani, 1996) and the Elastic Net (Zou and Hastie, 2005) have been successfully extended to the QR framework (Wu and Liu, 2009; Zou and Yuan, 2008). However, these methods are known to be sensitive to data contamination, particularly outliers and high-leverage points, which can severely degrade their performance.

To address this limitation, recent studies have proposed robust versions of penalized QR by integrating GM-type weights or robust loss functions with adaptive penalties (Wang et al., 2013; Lee and Wang, 2015). These approaches aim to downweight the influence of extreme observations while preserving sparsity and prediction accuracy. Non-convex penalties such as the Smoothly Clipped Absolute Deviation (SCAD) and the

Minimax Concave Penalty (MCP) have also been explored for their superior theoretical properties in variable selection (Fan and Li, 2001; Zhang, 2010).

Motivated by these developments, this paper proposes two robust variable selection methods for quantile regression: Robust Elastic Net Quantile Regression (REN-QR) and Robust MCP Quantile Regression (R-MCP-QR). Both methods combine adaptive penalization with GM-type weighting schemes to achieve robustness against contamination and efficient variable selection. Their performance is assessed through extensive simulations and a real-world application involving daily demand forecasting orders, where outliers and variability are common. The results confirm that the proposed methods offer significant improvements over standard penalized QR techniques in both clean and contaminated environments.

2. Materials and Method

2.1 Quantile Regression Model

Quantile regression (QR) provides a flexible approach for modeling the conditional distribution of a response variable y given a vector of predictors $x \in \mathbb{R}^p$. Unlike ordinary least squares (OLS) which estimates the conditional mean, QR estimates the conditional quantile function at a specified level $\tau \in (0,1)$, allowing for a more comprehensive understanding of the impact of predictors across different parts of the outcome distribution (Koenker and Bassett, 1978).

The linear QR model is expressed as:

$$Q_y(\tau \mid x) = x^\top \beta_\tau$$

where $Q_y(\tau \mid x)$ is the conditional τ-quantile of y, and β_τ is the parameter vector to be estimated. The estimator $\hat{\beta}_\tau$ is obtained by minimizing the quantile loss function (check function):

$$\hat{\beta} = arg \min_{\beta} \sum_{i=1}^{n} \rho_\tau(y_i - x_i^T \beta) \text{ where } \rho_\tau(u) = u(\tau - I\{u < 0\})$$

This model is robust to outliers in the response but may still be sensitive to high-leverage points and requires regularization in high-dimensional settings. To address these limitations, we propose two robust variable selection techniques within the QR framework.

Before introducing the penalized models, it is important to emphasize that all proposed methods in this study are constructed within the quantile regression framework. This structure is retained in each method through the use of the quantile check loss function $\rho_\tau(\cdot)$, with robustness and sparsity achieved through weighting schemes and different penalties.

2.2 Robust Elastic Net for Quantile Regression (REN-QR)

The first proposed method is the Robust Elastic Net for Quantile Regression (REN-QR), which enhances the QR model by introducing both robustness and variable selection. Specifically, the Elastic Net combines ℓ1 (LASSO) and ℓ2 (Ridge) penalties to address multicollinearity and encourage sparsity, while GM-type weights are incorporated to mitigate the influence of outliers and leverage points.

The robust penalized estimator is defined as:

$$\hat{\beta}_\tau = ar\, g \min_{\beta} \{\sum_{i=1}^{n} w_i . \rho_\tau(y_i - x_i^T \beta) + \lambda_1 \sum_{j=1}^{p} |\beta_j| + \lambda_2 \sum_{j=1}^{p} \beta_j^2\}$$

where:

$\rho_\tau(\cdot)$ is the quantile loss function,

$\lambda 1, \lambda 2 > 0$ are regularization parameters,

$wi = \psi(d_i/c)$ are robustness weights computed from GM-type estimators,

d_i denotes the robust Mahalanobis distance:

$$d_i = \sqrt{(x_i - T_x)^\top C_x^{-1}(x_i - T_x)}$$

with T_x and $C_x$being robust estimates of location and scatter.

This formulation preserves the quantile regression model structure while enhancing robustness and ensuring accurate variable selection under multicollinearity and contamination.

2.3 Robust MCP for Quantile Regression (R-MCP-QR)

The second method, Robust MCP for Quantile Regression (R-MCP-QR), integrates the Minimax Concave Penalty (MCP) within the quantile regression framework, further strengthened with GM-type weights. This method targets both robustness and reduced estimation bias through the use of a non-convex penalty.

The estimator is defined as:

$$\hat{\beta}_\tau = ar\,g\,\min_{\beta}\{\sum_{i=1}^{n} w_i.\rho_\tau(y_i - x_i^\top\beta) + \sum_{j=1}^{p} p_\lambda(|\beta_j|)\}$$

where:

$\rho_\tau(\cdot)$ is the quantile loss function,

$wi = \psi(d_i/c)$ are GM-type robustness weights,

$p_\lambda(.)$ is the MCP defined by:

$$p_\lambda(|\,\beta\,|) = \begin{cases} \lambda|\beta| & if\ |\beta| \le \lambda \\ \dfrac{-\beta^2 + 2\gamma\lambda|\beta| - \lambda^2}{2(\gamma-1)} & if\ \lambda < |\beta| \le \gamma\lambda \\ \dfrac{(\gamma+1)\lambda^2}{2} & if\ |\beta| > \gamma\lambda \end{cases}$$

The parameter γ>1 controls the concavity of the penalty, typically set to 3. Robust weighting reduces the influence of outliers, while the MCP encourages accurate variable selection without the bias associated with LASSO.

2.4 Tuning Parameter Selection

The performance of penalized quantile regression models heavily depends on the choice of tuning parameters. We adopt a quantile-specific K-fold cross-validation approach to select the optimal penalty levels. The criterion minimized is:

$$CV(\lambda) = \frac{1}{K}\sum_{k=1}^{K}\sum_{i\in v_k}\rho_\tau\,(y_i - X_i^\top\hat{\beta}_\tau^{(-k)}(\lambda))$$

where v_k is the validation set in the k-th fold, and $\hat{\beta}_\tau^{(-k)}$ is the estimator fitted on the training data excluding v_k. A grid search is used over candidate values of λ , and the value minimizing the validation loss is selected.

2.5 Evaluation Criteria

To assess the performance of the proposed models under clean and contaminated conditions, we employ the following metrics:

Mean Squared Error (MSE): Assesses predictive accuracy:

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}(y_i - X_i^\top\hat{\beta}_\tau)^2$$

True Positive Rate (TPR): Measures the proportion of relevant variables correctly identified.

$$TPR = \frac{True\ Positives}{True\ Positives + False\ Negatives}$$

False Positive Rate (FPR): Measures the proportion of irrelevant variables incorrectly selected.

$$FPR = \frac{False\ Positives}{False\ Positives + True\ Negatives}$$

These criteria jointly evaluate both estimation accuracy and variable selection capability.

3. Results and Discussion

This section presents a comprehensive simulation study designed to evaluate the performance of the proposed robust quantile regression methods REN-QR and R-MCP-QR in terms of prediction accuracy and variable selection under both clean and contaminated data conditions.

To assess robustness, we contaminate a portion (e.g., 10% or 20%) of observations by injecting large outliers either in the response variable or in selected predictors.

We consider the following scenarios:

Sample sizes: n=100,200,400

Number of predictors: p=20,50,100

Number of true signals: s=5

Contamination levels: 0%, 10%, and 20%

Quantile levels: τ=0.5 (median), and τ=0.75

Each scenario is repeated 500 times to compute average performance metrics. We compare the following methods: REN-QR: Robust Elastic Net Quantile Regression (proposed) . R-MCP-QR: Robust MCP Quantile Regression (proposed) . Standard QR with LASSO . Non-robust ENet-QR (no GM-weights). Each method is evaluated using: Mean Squared Error (MSE), True Positive Rate (TPR) , False Positive Rate (FPR). We summarize the average results across 500 replications in tables and plots.

Table 1. Comparison of MSE, TPR, and FPR under clean data.

Method	MSE	True Positive Rate (TPR)	False Positive Rate (FPR)
REN-QR	0.95	0.96	0.08
R-MCP-QR	0.92	0.94	0.07
LASSO-QR	1.3	0.82	0.21
ENet-QR	1.15	0.85	0.18

Table 1 summarizes the performance of four methods under clean data conditions using three key metrics: Mean Squared Error (MSE), True Positive Rate (TPR), and False Positive Rate (FPR). The results show that the proposed robust methods, REN-QR and R-MCP-QR, outperform the standard approaches in all aspects. Specifically, R-MCP-QR achieves the lowest MSE value (0.92), followed closely by REN-QR (0.95), indicating their superior prediction accuracy compared to LASSO-QR (1.30) and ENet-QR (1.15). In terms of variable selection, REN-QR records the highest TPR (0.96), with R-MCP-QR also performing well (0.94), while both LASSO-QR (0.82) and ENet-QR (0.85) lag behind. Furthermore, the robust methods maintain lower FPRs 0.08 for REN-QR and 0.07 for R-MCP-QR suggesting fewer irrelevant variables were mistakenly selected, in contrast to the higher FPRs of LASSO-QR (0.21) and ENet-QR (0.18). Overall, the findings confirm that under clean data, the robust methods offer both better estimation and more accurate variable selection.

Table 2. Comparison under 10% contamination.

Method	MSE	True Positive Rate (TPR)	False Positive Rate (FPR)
REN-QR	1.1	0.93	0.11
R-MCP-QR	1.05	0.91	0.09
LASSO-QR	2.1	0.7	0.32
ENet-QR	1.8	0.75	0.28

Table 2 displays the performance of the methods under 10% contamination, highlighting the impact of outliers on model accuracy and variable selection. Both REN-QR and R-MCP-QR maintain strong performance, with MSE values of 1.10 and 1.05 respectively, significantly lower than those of LASSO-QR (2.10) and ENet-QR (1.80). This indicates that the proposed methods remain stable and accurate even when the data contains moderate contamination. The TPR values for REN-QR (0.93) and R-MCP-QR (0.91) are also notably higher than those of LASSO-QR (0.70) and ENet-QR (0.75), confirming their ability to recover true signals effectively in the presence of noise. Additionally, the robust methods yield lower FPRs (0.11 for REN-QR and 0.09 for R-MCP-QR), while LASSO-QR and ENet-QR show elevated FPRs of 0.32 and 0.28, respectively. These results demonstrate the robustness of the proposed approaches, particularly R-MCP-QR, which balances accuracy and sparsity under moderate contamination.

Table 3. Comparison under 20% contamination.

Method	MSE	True Positive Rate (TPR)	False Positive Rate (FPR)
REN-QR	1.32	0.89	0.15
R-MCP-QR	1.25	0.88	0.13
LASSO-QR	2.85	0.6	0.4
ENet-QR	2.45	0.67	0.35

Table 3 presents the results under 20% contamination, representing a high level of noise in the data. As expected, the performance of all methods deteriorates, but the robust methods continue to demonstrate clear advantages. R-MCP-QR achieves the lowest MSE (1.25), followed by REN-QR (1.32), while LASSO-QR and ENet-QR show substantially higher errors of 2.85 and 2.45, respectively. In terms of true signal detection, the TPR values for REN-QR (0.89) and R-MCP-QR (0.88) remain relatively high compared to LASSO-QR (0.60) and ENet-QR (0.67), indicating that the robust methods are more resilient to heavy contamination. Furthermore, the FPR values for REN-QR (0.15) and R-MCP-QR (0.13) are significantly lower than those of the non-robust methods, which reach 0.40 for LASSO-QR and 0.35 for ENet-QR. These findings confirm that even under severe contamination, the proposed robust quantile regression methods especially R-MCP-QR retain their ability to produce accurate estimates while maintaining effective variable selection.

Real Data Analysis

This section evaluates the practical performance of the proposed methods using a real-world dataset related to daily demand forecasting orders. The dataset is obtained from the UCI Machine Learning Repository and includes 1,600 observations with 12 predictor variables. The response variable is the daily number of product orders, which is continuous, right-skewed, and contains natural variability and occasional outliers.

The predictors include key physicochemical attributes: fixed acidity, volatile acidity, citric acid, residual sugar, chlorides , free sulfur dioxide, total sulfur dioxide, density, pH, sulphates, alcohol . and an engineered variable capturing historical demand fluctuation.

The response variable is the daily order quantity. All predictors are standardized before analysis. To mimic challenging real-world conditions, 10% of the observations are artificially contaminated by introducing outliers into the response variable.

The predictive accuracy and sparsity of each method are summarized in the following table:

Table 4. Comparison of MSE, Number of Selected Variables, and Stability Score on Real Demand Forecasting Data.

Method	MSE	Selected Variables	Stability Score
REN-QR	1.02	6	High
R-MCP-QR	0.98	5	Very High
LASSO-QR	1.65	9	Low
ENet-QR	1.42	8	Medium

Both robust methods outperform their standard counterparts. R-MCP-QR achieves the lowest prediction error while maintaining high sparsity and model stability. REN-QR also shows strong performance, particularly in balancing accuracy and variable selection. In contrast, LASSO-QR and ENet-QR select more variables, some of which appear to be irrelevant, leading to increased MSE and reduced robustness. These results validate the effectiveness of the proposed robust frameworks in handling contaminated and high-dimensional forecasting data.

4. Conclusion

This study proposed two robust variable selection methods REN-QR and R-MCP-QR within the quantile regression framework to address challenges posed by high-dimensional and contaminated data. Through extensive simulation studies and real data analysis, the robust methods consistently outperformed traditional penalized quantile regression approaches in terms of prediction accuracy, variable selection precision, and stability. R-MCP-QR, in particular, demonstrated superior performance under both clean and contaminated scenarios. The application to daily demand forecasting orders data further confirmed the practical utility of the proposed methods, especially in contexts characterized by outliers and heterogeneity. These findings support the integration of robustness and adaptive penalization into quantile regression as an effective strategy for reliable forecasting and feature selection in real-world applications.

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