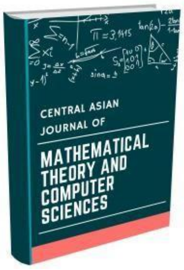

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Article

Calculating The Probability of Events Using Logical Operations

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Abstract: General Background: The merge between probability theory and mathematical logic introduces a useful analytical tool of uncertain events modeling in systems whose operations are described by logical conditions. Particular Background: Probabilistic dependencies are closely related to logical operations at conjunction and disjunction in relay-contact schemes and in electronic systems, which have a practical use in both engineering and teaching. Knowledge Gap: Nevermore, there has been low priority on the use of both logical operations and event analysis using probabilistic rules, specifically in teaching case. Objective: The objective of the following article is to find out how to use logical operations to figure out the likelihood of events and to investigate their application to circuit-based situations. Results: Based on classical probabilities and logical truth tables, the article discusses an illustration in which there are several connectors through which energy can flow. It concludes that in 7 out of 16 combinations the event will happen, the complementary logic of which confirms the conclusion. Further the distribution law of the working time of a lamp is calculated depending on the difference equations. Novelty: The treatment is a composition of probabilistic assessment and logic-based model. Implications: This technique has theoretical and practical relevance in the teaching of probability and examination of system reliability.

Keywords: relay-contact schemes, logic $\{\neg, \wedge, \vee, \rightarrow, \leftrightarrow\}$ operations, parallel and consecutively connect principles, phenomenon, reflection

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1. Introduction

Two major subsets of mathematics that are important in modelling uncertainty and formal reasoning are probability theory and mathematical logic. The study of randomness is large-scale, as the probability theory finds wide application in many fields, including natural sciences, engineering, and the social behavior sector[1]. Equally, formal verification, artificial intelligence and digital circuit design rely on mathematical logic[2]. They have interconnected conceptual roots, the study of which is just a different well-established discipline, although related, when they are applied to binary systems (decision problems), deciding how the systems, and subsets of both disciplines, can behave[3].

Applications of all these fields are especially pertinent to the study of the operation of relay-contact schemes and other logical systems whose operations are frequently binary: the element performs or fails to perform. Logical functions Logical operation Logical operations (conjunction, disjunction and negation), correspond to series and parallel circuits devices, which are typically used in electrical engineering. Rew set over to glancing at it through the prism of logical actions, we shall be able to calculate our chances of a success or failure in a sane and computationally expedient method[4].

Even though probability and logic have a natural synergy, they appear in educational and practical sources as independent. This leaves a knowledge gap in having an idea of how logical framework can be involved to simplify calculation of probabilities during the design and reliability analysis of systems[5]. The works by A.N. Kolmogorov and I. NeMatov make emphases on the theoretical preparation, whereas applied integration (in particular, with the help of truth tables and schematics) remains underused in practice and education[6].

The approach to this research is a descriptive-analytical technique; first, classical rules of probability are used, then are applied to systems that can be modeled using logical operations. All possible states of elements in the system are then pictured with the help of truth tables, and the classical probability definition that is based on frequencies will be used to obtain the likelihoods of their occurrences[7]. Furthermore, a practical application, which could be described as exponential distribution of operating time of an electronic lamp, demonstrates how to use the differential equation in order to determine probability density function. These are the ways that seek to prove the usefulness of logical thinking to overcome the issues of probabilities[8].

The anticipated result of the study is two-fold. First, it demonstrates to show that logical modeling does provide a legitimate and methodic methodology to address the problems that were only dealt with by probability theory. Second, the results elicit a need to make a combination of these disciplines in order to increase the efficiency of teaching and analysis of the system[9]. These implications have an effect in both classroom levels- where clarity and inter-disciplinary transfer can promote student learning- and in more practical applications such as engineering, which believes that system reliability can be measured with logical instruments. Therefore, the paper has a combination of both theory and practical value in linking use of logic based reasoning and probability computation[10].

2. Materials and Methods

The methodological approach of this study is based on the comparative application of classical probability theory and mathematical logic to analyze and calculate the probability of events occurring in technical systems. The investigation begins with the formulation of a representative problem involving a relay-contact scheme, where the passage of current from one terminal to another depends on the configuration of multiple connectors. The event of interest—current successfully passing through the system—is defined in probabilistic terms using the classical formula P(A)=mnP(A) = \frac{m}{n}P(A)=nm, where mmm represents the number of favorable outcomes and nnn the total number of possible outcomes. These outcomes are systematically enumerated using logical structures, and corresponding probabilities are calculated. To mirror this process through logical reasoning, the same scheme is translated into a logical expression involving conjunctions and disjunctions, which represent serial and parallel connections respectively. A truth table is constructed to determine which logical configurations result in successful current flow, thus providing a logical analog to the probabilistic computation. Additionally, the methodology includes a continuous probability analysis using a real-world example—the failure time of an electronic lamp. This part of the study uses an exponential distribution model and solves a differential equation to derive the probability density function. The integration constant is determined using boundary conditions drawn from empirical data. By combining logical reasoning tools with probabilistic models, the methodology offers a dual perspective that enhances accuracy and pedagogical clarity in probability-based system analysis.

3. Results and Discussion

Neighbor theories study students reading efficiency increases , complete and solid knowledge to receive help gives [11].

This in the sense probabilities theory elements with mathematician logic theory elements between some commonalities there is is , this article this to the topic is dedicated.

About it probabilities theory and mathematician statistics considered the " father " of science academic ANKolmogorov thoughts There are possibilities . theory and mathematician logic theory mutual close concepts existence about information Also , BAKordensky Probabilities are also in the book theory elements in general , to them dedicated relay-contact schemes to be given about information given . I.Ne'matov in the book logical $\{\neg, \wedge, \vee, \rightarrow, \leftrightarrow\}$ actions connection to the schemes applications , parallel and consecutively connect principles given[12] .

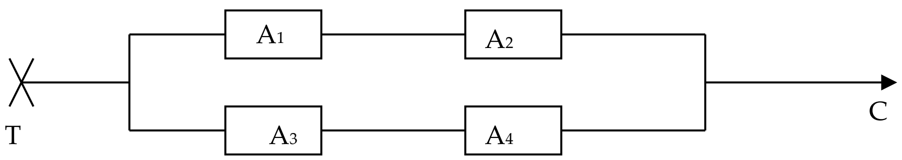
Probabilities theory events ABC… letters with is determined by the event A. probability With P(A) is determined and that probability is between 0 and 1 $0 \leq P(A) \leq 1$ It is considered that . Occurs to give possible not been with event V is determined , its probability P (V)=0 is inevitable with the event is given , its with probability P(V)=1 is given . So , every how of the event probability of 1 big not , from 0 small not , P(A) is positive and measurable is a function .

Such proportionality mathematician logic It also exists in the theory of probability . the concept of " event " in the theory mathematician logic to the concept of " reflection " in the theory adaptation possible . Reflection concept mathematician logic theory elementary concept are the letters A ,B,C ,… with is determined . Every one The values 1-true and 0-false are assigned to the argument. suitable as is put . Other as in other words , every how reasoning { 0,1 }={ ë , p } two elemental from the collection value acceptance does [13].

Probabilities theory and mathematician logic theory here this thoughts to say possible

Figure 1 below illustrates one issue probability theory rules with and mathematician logic rules also solve with methods We'll see .

Figure 1. Two par meters consecutively and parallel connection



scheme Given . From T to C energy passes through , A_1, A_2, A_3, A_4 - connectors vine to pass $\oplus$ and vine not to pass … let's say ,

Solution 1. Transfer the current from T to C transition probability find need let it be , that is which in cases scheme performance determination need[14] .

With A current from T to C transition the event If we define P(A) find need , or, what P(A)-? Classical probability from the formula $P(A) = \frac{m}{n}, \quad 1 \leq m \leq n$ we use , n common opportunities number , m- us satisfactory opportunities number . General opportunities number $N = 2^4 = 16$ will be , us satisfactory opportunities number (m) from the following " matrices" determination possible :

$\left(\begin{array}{c} ++ \\ \hline -+ \end{array}\right)$, $\left(\begin{array}{c} ++ \\ \hline +- \end{array}\right)$, $\left(\begin{array}{c} ++ \\ \hline -- \end{array}\right)$ and again their ways if we replace ,

$\left(\begin{array}{c} -+ \\ \hline ++ \end{array}\right)$, $\left(\begin{array}{c} +- \\ \hline ++ \end{array}\right)$, $\left(\begin{array}{c} -- \\ \hline ++ \end{array}\right)$, finally $\left(\begin{array}{c} ++ \\ \hline ++ \end{array}\right)$ divided by m= 7 equal

The rest in cases vine It won't pass .

So , $P(A)=\frac{m}{n}=\frac{3+3+1}{16}=\frac{7}{16}$ A- current not to pass event If we say that P (A)+P(A)=1 for $P(A)=\frac{9}{16}$ will be .

Solution 2. Transfer the current from T to C the passage mathematician logic deeds with solution Let's do it .

Logic deeds definition and them relay-contact to the schemes from their implementation as is known consecutively connect principle logical from deeds conjunction gives it , it is marked with the " $\wedge$" symbol with given ; parallel connection principle and disjunction gives it , it is marked with the symbol " $\vee$". with If there is a complex " connection " , it is given mixture scheme suitable comes to the scheme (1) suitable logical formula $U(A_1,A_2,A_3,A_4)=(A_1\wedge A_2)(A_3\wedge A_4)$ in the form of will be . this to the formula suitable truth table We will make it . Comments There are 4 of them. for general opportunities number $2^4=16$ It will be[15] .

In order to analyze the probability of current passing through the logical scheme using Boolean operations, a comprehensive truth table was constructed based on all possible states of the connectors. Table 1 presents all 16 possible combinations of binary input values and identifies the corresponding outcomes where the circuit either conducts (1) or fails to conduct (0) current from T to C.

Table 1. Truth Table for Determining Circuit Functionality Based on Logical States of Relay-Contact Connectors

A_1	A_2	A_3	A_4	$A_1\wedge A_2$	$A_3\wedge A_4$	$(A_1\wedge A_2)\vee$ $(A_3\wedge A_4)$
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	1	0	0	0	0
0	0	1	1	0	1	1
0	1	0	0	0	0	0
0	1	0	1	0	0	0
0	1	1	0	0	0	0
0	1	1	1	0	1	1
1	0	0	0	0	0	0
1	0	0	1	0	0	0
1	0	1	0	0	0	0
1	0	1	1	0	1	1
1	1	0	0	1	0	1
1	1	0	1	1	0	1
1	1	1	0	1	0	1
1	1	1	1	1	1	1

Truth 1 in the table s scheme worked " vine" "passed" , and 0 not working vine did not pass indicates . From the table It seems that out of 16 cases , 7 are 1 and 9 are 0, so in 7 cases for vine passes , in 9 cases vine It won't pass .

This is math. logic deeds using probability theory issues is the solution .

Example : Electronics of the lamp work time X random quantity Let it be . Working of the lamp how much big not been time inside from work exit probability his/her next work on time approximately proportional that and of the lamp before how much because it worked related not to be experience using determined . X is random . of the amount distribution function We find . The lamp A event that runs for at least X days his/her x and $x+\Delta x$ days between from work output and event B Let it be .

$$P(A)=P(X\geq x)=1-P(X<x)=1-F(x) \qquad (1)$$
$$P(B)=P(x<X \quad x+\Delta x)=F(x+\Delta x)-F(x)$$

Working the lamp observe we started for experience as a result $F(B)$ perhaps it's not maybe $P_A(B)$ conditional perhaps is determined . In that case multiplication theorem as a result we use .

$$P(A)\cdot P_A(B)=P(B)\cdot P_B(A) \quad (2)$$

if event B face gave If , A is an event face to give $P_B(A)=1$ That's why for

$$P_A(B)=\frac{P(B)}{P(A)}=\frac{F(x+\Delta x)-F(x)}{1-F(x)} \quad (3)$$

if Δx enough small if ,

$$P_A(B)\approx\frac{\phi(x)\Delta x}{1-F(x)}(4)$$

$\varphi(x)=F'(x)-$ distribution density second from the side on condition according to $P_A(B)$ perhaps for

$$P_A(B)\approx k\Delta x(5)$$

(1) and (2) are approximate equalities by comparison and their mistakes $\Delta x\to 0$ to zero at aspiration

$$\frac{\varphi(x)}{1-F(x)}=k \text{ or } \varphi(x)=k-kF(x) \ (6)$$

Last PCB differentially , being sought probabilities density for

$$\varphi'(x)=k\varphi(x)$$

differential equation harvest We do this equation . integration and $\varphi(x)=Ce^{-kx}$

(C- optional (unchangeable) solution gives . To find C in our case $x\geq 0$ since it was $x<0$ at $\varphi(x)\equiv 0$ that it is record we will do .

$$\varphi(x)=\begin{cases}x<0 \quad bo'lsa, \quad 0\\ x\geq 0 \quad bo'lsa \quad ke^{-kx}\end{cases} \qquad (7)$$

$$F(x)=\begin{cases}x<0 \quad bo'lsa, \quad 0\\ x\geq 0 \quad bo'lsa \quad 1-e^{-kx}\end{cases}$$

k coefficient experience using discovery It needs to be 0.01 . equal to (time) days with when you are in a hurry).

1. Electronic lamp 30 days of work on the key probability
2. His 30th and 40th days between disruption probability we count

First to the question $P_1=1-F(30)=1-\left(1-e^{-0,3}\right)=e^{-0,3}\approx 0,74$, second perhaps for and , $p_2=\int_{30}^{40}\varphi(x)dx=F(40)-F(30)=e^{-0,3}-e^{+0,4}\approx 0,74-0,67=0,07$.

4. Conclusion

To sum up, the given paper shows that logical operations like conjunction or disjunction can be successfully used to calculate the probability of events in the structured systems like relay-contact schemes, which proves that there is a meaningful relation between the theory of probability and mathematical logic. It has been revealed by the analysis that logical modeling, truth tables and logical expressions combined with classical probability formulas produce consistent and meaningful results, which could be witnessed in the calculated probability of current passing through a given scheme. Moreover, the application of differential equations to the probability density of the life time of a lamp explains how the continuous random variables may be used in the logical-probabilistic modeling. All this suggests that combining formal reasoning with probabilistic calculation does not only result in tools of greater value when it comes to precision in analysis efforts but it also yields useful classroom teaching resources when presenting mathematical modeling of systems and system reliability. The method has a great deal of prospect in application in engineering and educational practices. Future investigations on the extension of these techniques to even more involved circuit configurations and digital systems, or in the creation of educational tools and software making use of the logical structures to educate probability in a more participatory and naturalistic format, are also to be developed.

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