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Article

Mathematical Periodontium model with Holling Type I: Chaos and Control

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Abstract: The periodontium is a assisting structure that surrounds and helps the teeth, it consists of various tissues consisting of the gingiva, the cementum, the periodontal ligament and alveolar helping bone. Knowing the reality that the periodontium is a complex system in the frame, this paper demonstrates viable mirrored image of chaos idea and the sector of periodontology, and most generally used practical responses to describe the average feeding rate of a predator are Lotka-Volterra type and Holling type practical reaction characteristic's.System features are discussed by its equilibrium points, stability, dissipativity and bifurcation analysis. Graphical representation through numerical simulations are presented. Our study has shown that the periodontium system (1) is bifurcate and unstable system. It has dissipative equilibrium point and conservative equilibrium point, in addition to all of this the periodontium system (1) indicates a state of extreme chaos with Lyapunov dimension $D_L = 1.7022130$ where the Lyapunov exponents are $L_1 = 0.908835$ *and* $L_2 = -1.294244$, after that we applied active control technique and adaptive control technique. By understanding the periodontium system (1) very well we estimated the rate of periodontitis in mathematical periodontium model 'b' as control parameter and successfully controlled the chaos. This brings us to the purpose of this research paper, whereas understanding the periodontium system's structure and function may prove valuable in managing illness.

Keywords: Chaos, Control techniques, Equilibrium points, Holling type, Periodontium system, stability analysis.

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1. Introduction

Population ecology is the examine of populations mainly populace abundance and how they alternate through the years. The interplay of populace in ecology can divide in to distinctive classes inclusive of mutualism, commensalism, opposition, prey-predator and etc. [1]. The interplay conduct among prey and predator in an atmosphere can purpose the country of the population to exchange. These interactions can have a positive and terrible effect, or maybe haven't any effect at the interacting species. So that what cause one species to end up extinct is a very excessive predation rate of the prey and the low increase charge of prey [2]. Most typically used functional responses to describe the average feeding charge of a predator are Lotka-Volterra type and Holling type purposeful reaction capabilities [3]. A predator's per capita feeding charge on prey, or its practical response, presents a foundation for predator–prey idea [4] As a unique "prey-predator machine "the periodontium is a set of interacting factors. Studies inside the 1960s gave experimental proof stressing the critical role of micro organism inside the etiology of periodontitis. The linear model in [5] implicated that bacterial deposits are a primary and crucial issue within the pathogenesis of periodontal sicknesses. Periodontal illnesses are a sort of gum disorder due to the extreme colonization of a bacteria. The commonplace signs and symptoms are with extra gum swelling and pain, which might also even harm the jaw bones and corresponding nerves and tissues. Gingivitis and periodontitis are the most commonplace bacterial infection within the human craniofacial vicinity. The sedimentation of plaque on the floor of tooth results in gingivitis due to abnormal and incorrect dental care. When untreated early, it may emerge to irreversible periodontal illnesses because of secretion of harmful pollutants [6]. To display the relevance of mathematical version to fitness troubles, it has been applied [7] to experimental oral biofilm initiation of 3 exclusive species: Porphyromonas gingivalis, Streptococcus gordonii and Terponema denticola. These species are worried in periodontal biofilms that could cause enamel/bone loss. A colonization hierarchy is installed from the beginning of oral biofilm growth in oral biofilms, which starts with number one colonizers (streptococcus), then secondary colonizers which includes fusobacterium, and eventually ends with the incorporation of anaerobic gram-bad pathogens, chargeable for periodontal diseases such as P.Gingivalis and T.Denticola. The attachment and improvement of pathogens inside the oral biofilm is therefore dependent on the attachment of number one and secondary colonizers [8], [9], [10]. Analyzing such interactions may be difficult, therefore mathematical modelling of organic approaches targets to translate conceptual hypotheses into the handiest feasible equations to benefit insight into fundamental mechanisms. So, this paper is dependent as follows:Section 2 presents a description of the periodontium system. Section 3 investigates the features of the periodontium system using equilibrium points, stability analysis, and dissipative properties [11][12][13][14][15][16]. Section 4 performs numerical and graphical analysis [17][18]. Section 5 applies chaotic periodontium system stabilization and displays its system dynamics [19][20][21][22]. Section 6 presents comparison tables before and after control. Finally, the study concludes

2. Materials and Methods

This study adopts a mathematical modeling approach to analyze the dynamic behavior of the periodontium as a prey–predator system. The system is defined using a Lotka-Volterra differential framework with Holling Type I functional response to describe the interaction between gingiva (prey) and periodontitis (predator). The model parameters were initialized based on standard biological assumptions, and stability was analyzed using equilibrium point analysis, Jacobian matrix evaluation, Routh-Hurwitz stability criteria, and Lyapunov functions. Numerical simulations were conducted in MATLAB to visualize phase portraits, bifurcation diagrams, and Lyapunov exponent dynamics. Subsequently, control strategies were introduced: (1) an adaptive control law incorporating biological parameter estimation and (2) an active nonlinear feedback control to stabilize system trajectories. Each method's effectiveness was evaluated by comparing the system's stability and chaos levels before and after control.

3. Results and Discussion

System description

A continuous time Lotka-Voltera prey-predator machine [8] wherein the competition between two species G and P is modeled by the subsequent normal differential equations

$$\left.\begin{aligned}\frac{dg}{dt} &= ag(1-g) - bgp \\ \frac{dp}{dt} &= cgp\end{aligned}\right\} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots..(1)$$

Where $g = g(t),\ p = p(t)$ denote the population density of two species at time t $and\ \ a\ ,b\ ,and\ c$ are positive parameters.

g from the word gingiva denote to the prey density in year (generation) n.

p from the word periodontitis denote to the predator density in year (generation) n.

bg represents the number of prey individuals consumed per unit time by an individual predator.

cgp is the predator response.

The parameters values are taken as:

$$a = 4 \quad , \quad b = 0.2 \quad , \quad c = 3.5 \} \dots\dots\dots\dots\dots\dots (2)$$

The kind I reaction is the result of the simple assumption that the probability of a given predator encountering prey in a hard and fast time c programming language T_t inside a set spatial place relies upon linearly on the prey density. Using the notation of Holling's paper [9] we might also express this dating in the shape

$$Z = bT_s g \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (3)$$

Where Z is the number of preys consumed by means of one predator, g is the prey density, T_s is the time available for looking and b is a consistent of proportionality termed the "discovery fee" by means of Holling. In the absence of a need to spend time handling the prey [10], all of the time can be used for looking i.e. $T_s = T_t$.

We have type I response assuming that the predator density p act independently, in a time $T_s = T_t$, the total number of preys will be reduced by quantity $bTgp$

System analysis

This section, analyze system (1) and highlight its features

Points of Equilibrium

The beginning in analyzing any system is to find points of its equilibrium, so we need to determine: $\frac{dg}{dt} = \dot{G} = 0 \ and \ \frac{dp}{dt} = \dot{P} = 0$. we solve for $g \ and \ p$.

1. $ag(1-g) - bgp = 0$ this simplifies to $g\,[a(1-g) - bp] = 0$
so, we have two possibilities: $g = 0 \quad or \quad a(1-g) = bp$
2. $cgp = 0$ this simplifies to $g = 0 \quad or \quad p = 0$

From the above equations, we get equilibrium points as follows:

When $p = 0$ then $ag(1-g) = 0$ and this simplifies to $g = 0 \quad or \quad g = 1$

So, we have:

(i) . The trivial stage or the extinction of all population equilibrium point: $E_0 = (G, P) = (0,0)$.

(ii). The extinction of predator equilibrium point: $E_1 = (G, P) = (1,0)$.

When $g = 0$ then p can be any value so, we have:

(iii). The extinction of prey equilibrium points which is also a line equilibrium point: $E_2 = (G, P) = (0, p)$

Stability analysis

the Jacobean matrix for periodontium system (1) is:

$$J = \begin{bmatrix} a - 2ag - bp & -bg \\ cp & cg \end{bmatrix}$$

By putting det $(J - K\mathrm{I}) = 0$, the characteristic equation for periodontium system (1) is:

$$k^2 + k(bp + 2ag - a - cg) + acg(1 - 2g) = 0 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (4)$$

Roots of characteristic equation

The balance of equilibrium points of periodontium gadget (1) is determined by using signal of the actual part of the Jacobean matrix's eigenvalues (function equation roots). If all the eigenvalues have poor actual parts then the gadget is strong. If at the least one eigenvalue has high quality actual part then it's far risky.

The characteristic equation at $E_0 = (0,0)$ is $k^2 - ak = 0 \dots\dots\dots\dots\dots\dots\dots\dots\dots (5)$

And the roots are $k_1 = 0 \ \ and \ \ k_2 = 4 > 0$

So, the periodontium system (1) is unstable.

The characteristic equation at $E_1 = (1,0)$ is $k^2 + k(a - c) - ac = 0 \dots\dots\dots\dots\dots\dots (6)$

And the roots are $k_1 = -4 \ \ and \ \ k_2 = 3.5 > 0$

So, the periodontium system (1) is unstable.

The characteristic equation at $E_2 = (0, P)$ is $k^2 + k(bp - a) = 0 \dots\dots\dots\dots\dots\dots\dots\dots (7)$

And the roots are $k_1 = 0 \quad and \quad k_2 = 4 - 0.2p$

So, the periodontium system (1) will be unstable when $p \le 20$.

Routh stability criteria

Theorem (i) the essential situation for the nonlinear device to be solid is: all the elements in the first column of Routh array must have nice values.

(ii) If a 0 is present on the primary column of Routh array, then the corresponding pole will oscillate on the imaginary axis.

(iii) the variety of times of signs and symptoms trade of the factors of first column is the wide variety of poles at the proper-hand aspect of s-plan which pressure the nonlinear system to be volatile.

From (5) we have

$a_2 = 1\,,\ a_1 = -4\,,\ a_0 = 0$

$$b_{n-2} = \frac{a_{n-1}a_{n-2} - a_n a_{n-3}}{a_{n-1}} \quad \Longrightarrow \quad b_0 = a_0 = 0$$

Table 1. Routh array at E_0

k^2	1	0
k^1	-4	0
k^0	0	0

From (6) we have

$a_2 = 1\,,\ a_1 = 0.5\,,\ a_0 = -14$

$$b_{n-2} = \frac{a_{n-1}a_{n-2} - a_n a_{n-3}}{a_{n-1}} \quad \Longrightarrow \quad b_0 = a_0 = -14$$

Table 2. Routh array at E_1

k^2	1	-14
k^1	0.5	0
k^0	-14	0

From (7) we have

$a_2 = 1 \quad ,\ a_1 = 0.2p - 4 \quad ,\ a_0 = 0$

$$b_{n-2} = \frac{a_{n-1}a_{n-2} - a_n a_{n-3}}{a_{n-1}} \quad \Longrightarrow \quad b_0 = a_0 = 0$$

Table 3. Routh array at E_2

k^2	1	0
k^1	0.2p - 4	0
k^0	0	0

Proposition 1:

From the calculated arrays 1 and 2, certainly, all of the signs of the first column are change and as a result, the periodontium system (1) is unstable at E_0 *and* E_1

For no sign changes in the first column of array 3, it is necessary that the condition $p \ge 20$ be satisfied. Thus, the characteristic equation (7) has roots with negative real parts if $p \le 20$. Otherwise the periodontium system (1) will be instability.

Hurwitz stability criteria

The Hurwitz determinants Δ_1 *and* Δ_2 of the characteristic equation are defined to be the minors of the characteristic equation. If all these minors hold positive values the system remains stable otherwise exhibits instability.

From equation (5) which is $k^2 - 4k = 0$ we find
$\Delta_1 = a_{n-1} = a_1 = -4 \quad < 0$

$$\Delta_2 = \begin{vmatrix} a_{n-1} & a_{n-3} \\ a_n & a_{n-2} \end{vmatrix} = \begin{vmatrix} a_1 & 0 \\ a_2 & a_0 \end{vmatrix} = a_1 a_0 = 0$$

From equation (6) which is $k^2 + 0.5k - 14 = 0$ we find
$\Delta_1 = a_{n-1} = a_1 = 0.5 \quad > 0$

$$\Delta_2 = \begin{vmatrix} a_{n-1} & a_{n-3} \\ a_n & a_{n-2} \end{vmatrix} = \begin{vmatrix} a_1 & 0 \\ a_2 & a_0 \end{vmatrix} = a_1 a_0 = -7 \quad < 0$$

From equation (7) which is $k^2 + k(0.2p - 4) = 0$ we find
$\Delta_1 = a_{n-1} = a_1 = 0.2y - 4$

$$\Delta_2 = \begin{vmatrix} a_{n-1} & a_{n-3} \\ a_n & a_{n-2} \end{vmatrix} = \begin{vmatrix} a_1 & 0 \\ a_2 & a_0 \end{vmatrix} = a_1 a_0 = 0$$

In this case we will have negative small minors when $p \leq 20$. As a result, we have the following proposition

Proposition 2:

From the solution above at each of equilibrium points that we have, since the values of one minor at least is less than zero therefore, the periodontium system (1) exhibits instability

Lyapunov Function

Let the Lyapunov function equation of system (1) as following:

$v(G,P) = \frac{1}{2}(g^2 + p^2) \quad > 0 \quad when \;\; G,P \; > 0$

By differentiating $v(G,P)$ we get:

$\dot{v}(G,P) \; = \; g \,.\, \dot{g} + p \,.\dot{p} = \;\; ag^2(1-g) - bg^2p + cgp^2$

If $\dot{v}(G,P) \; < 0$ then the periodontium system (1) is stable. This is the stability of periodontium system (1) by Lyapunov function. Now, we test the stability at equilibrium points:

$v(0,0) = 0 \; and \; \dot{v}(0,0) = 0$ so, the system is unstable at $E_0 = (0,0)$

$v(1,0) = \frac{1}{2}g^2 \;\; and \; \dot{v}(1,0) = 0$ so, the system is unstable at $E_1 = (1,0)$

$v(0,p) = \frac{1}{2}p^2 \;\; and \; \dot{v}(0,p) = 0$ so, the system is unstable at $E_2 = (0,P)$

Dissipative

Let $f_1 = \frac{dg}{dt} \;\; and \;\; f_2 = \frac{dp}{dt}$

$F = \frac{\partial f_1}{\partial g} + \frac{\partial f_2}{\partial p} = a - 2ag - bp + cg = a - bp + (c - 2a)g$

Notice that dissipative does not depend on the parameters only but it also depends on the state variables. The periodontium system (1) is dissipative and We can prove our word through trace matrix as following:

$\sum \mathrm{Tr(J)} = \; \nabla.(f_1, f_2)^T \; = \; \frac{\partial \mathrm{f}_1}{\partial g} + \frac{\partial \mathrm{f}_2}{\partial \mathrm{p}} = a - bp + (c - 2a)g = F.$

By substitute equilibrium points in F , we get the below result

proposition 3: according to equilibrium points in we find that:

at $E_0 = (0,0) \;\; then \; F = 4 \; > 0$ and the periodontium system (1) is conservative

at $E_1 = (1,0) \; then \; F = -0.5 \; < 0$ and the periodontium system (1) is dissipative

at $E_2 = (0,P) \; then \; F = 4 - 0.2p$ and the periodontium system (1) will be dissipative when $p \geq 20$ otherwise, it will be conservative.

NUMERICAL AND GRAPHICAL ANALYSIS

In this phase, we are able to use numerical simulation to affirm the preceding theoretical results and show the dynamic conduct of the chaotic periodontium gadget (1)

Trajectories and State-Space

The trajectories for gingiva and periodontitis will be simulated at initial values $[G_0, P_0] = [0.000001\,, 0.000001]$ and figure1 shows it. While the State-Space of periodontium system (1) will be simulated at initial condition $[G_0, P_0] = [0.00005\,, 0.00005]$ as shown in figure2.

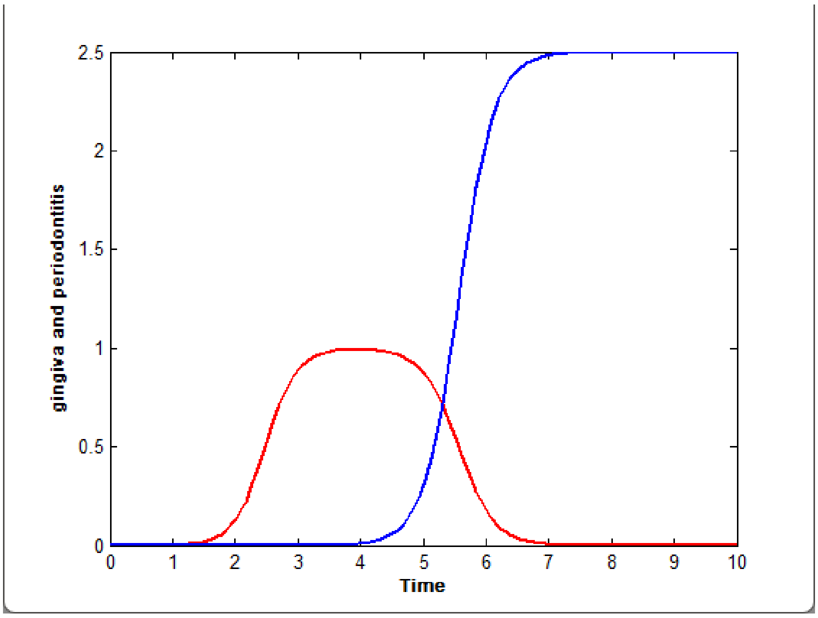


Figure 1. Trajectories of periodontium system (1): gingiva and periodontitis over time

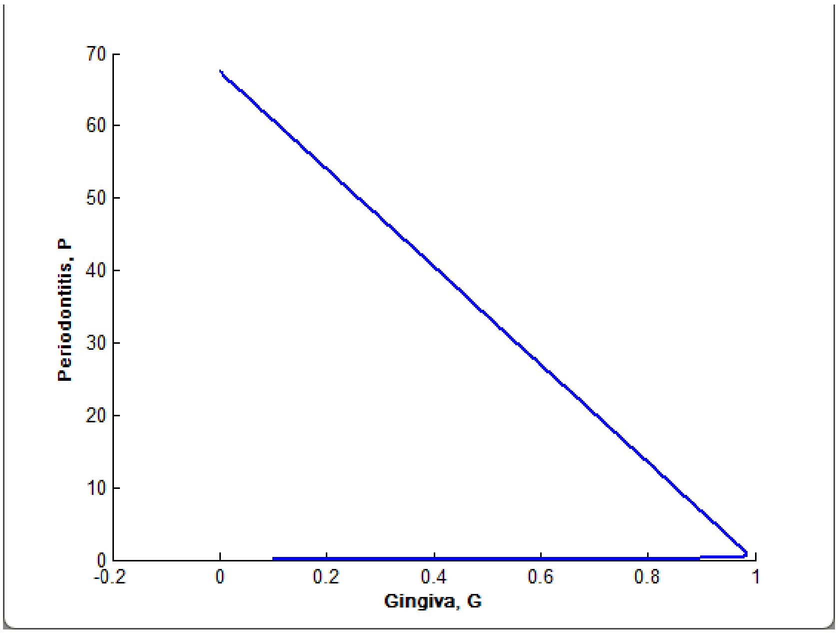


Figure 2. State-space for periodontium system (1): periodontitis versus gingiva

Bifurcation Diagram

Firstly, we will vary the rate of periodontitis in mathematical periodontium model ‘b’ in order to observe the bifurcate in gingiva. The bifurcation parameter range at initial condition [0.001, 0.001] will be taken at line space (0.0001, 12, 150) for (60) time span. Secondly, we will vary the periodontitis parameter ‘c’ at initial condition [0.5, 0.5] and the bifurcation parameter range will be taken at line space (1, 5, 300).lastly we will vary the growth rate of gingiva ‘a’ at initial condition [1, 1] and the bifurcation parameter range will be taken at line space (3, 6, 200) for (80) time span. The bifurcation diagram for periodontium system (1) are shown in figure3, figure4 and figure5 respectively.

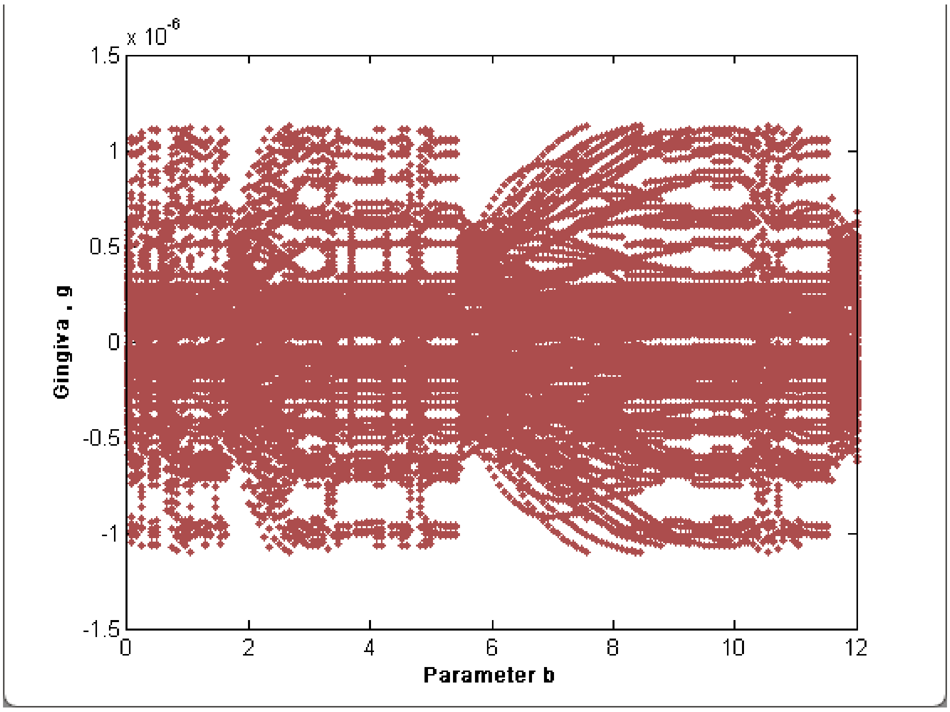


Figure 3. Bifurcation diagram for gingiva versus parameter b

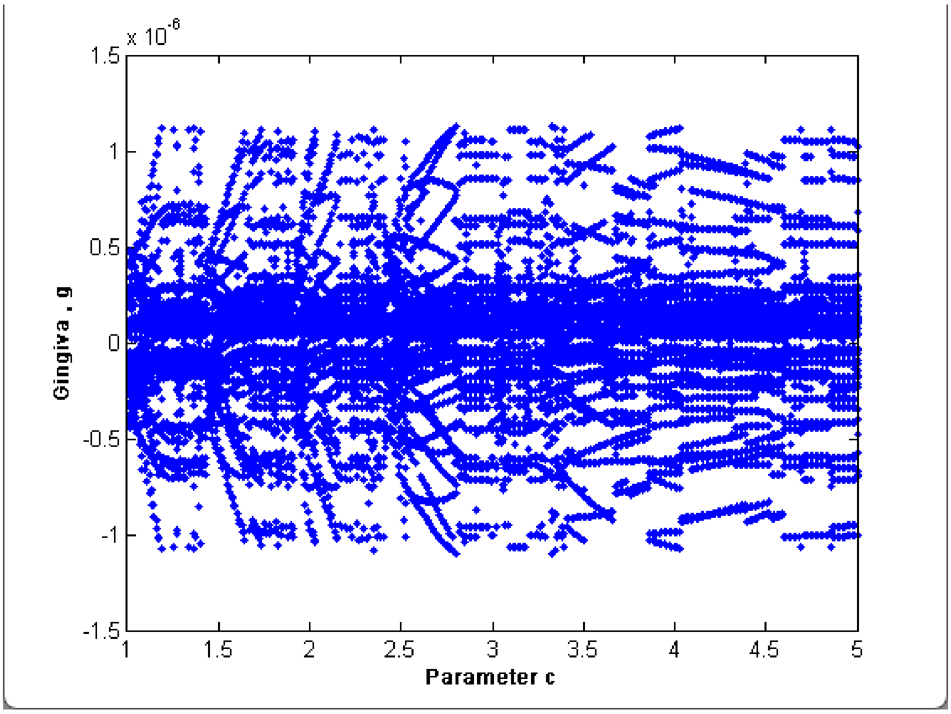


Figure 4. Bifurcation diagram for gingiva versus parameter c

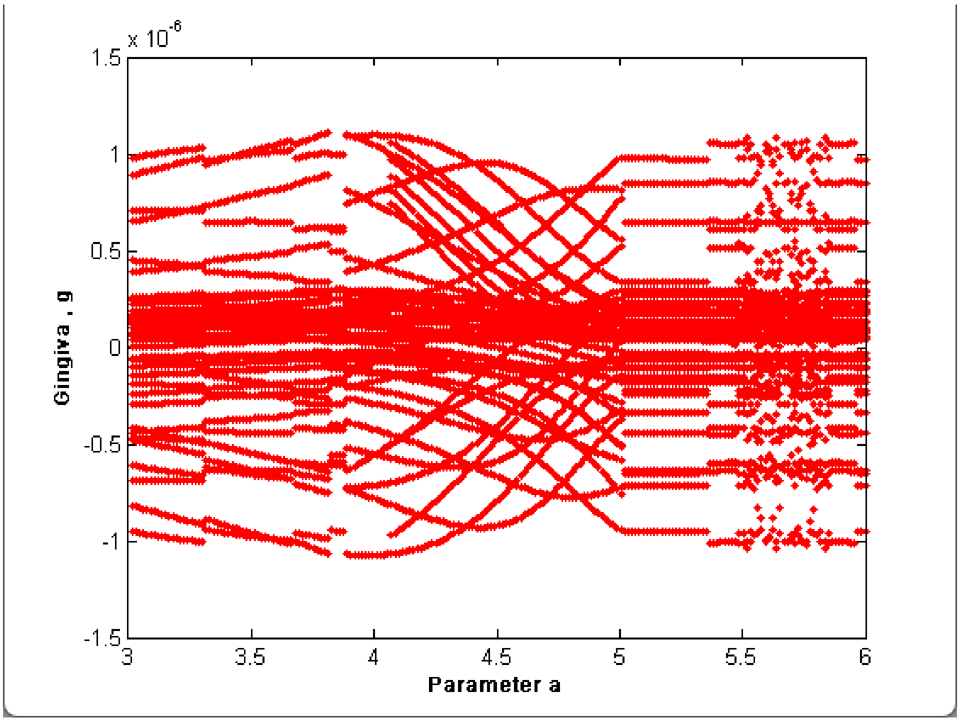


Figure 5. Bifurcation diagram for gingiva versus parameter a

Lyapunov Exponent and Lyapunov Dimension

According to the nonlinear chaos concept, the Lyapunov exponents degree the exponential costs of divergence and convergence of nearby trajectories in state space of the gadget.Thus, according to the parameters values in equation (2) and let the initial value $[G_0, P_0] = [0.1\,, 0.00005]$ the corresponding Lyapunov exponents for the periodontium system (1) are:

$L_1 = 0.908835 \quad and \quad L_2 = -1.294244$

Therefore, the Lyapunov Dimension of this system is:

$$D_L = 1 + \frac{L_1}{|L_2|} = 1.7022130$$

Thus, as shown in figure 6 the dynamics of Lyapunov exponents, we illustrate that the periodontium system (1) is chaotic.

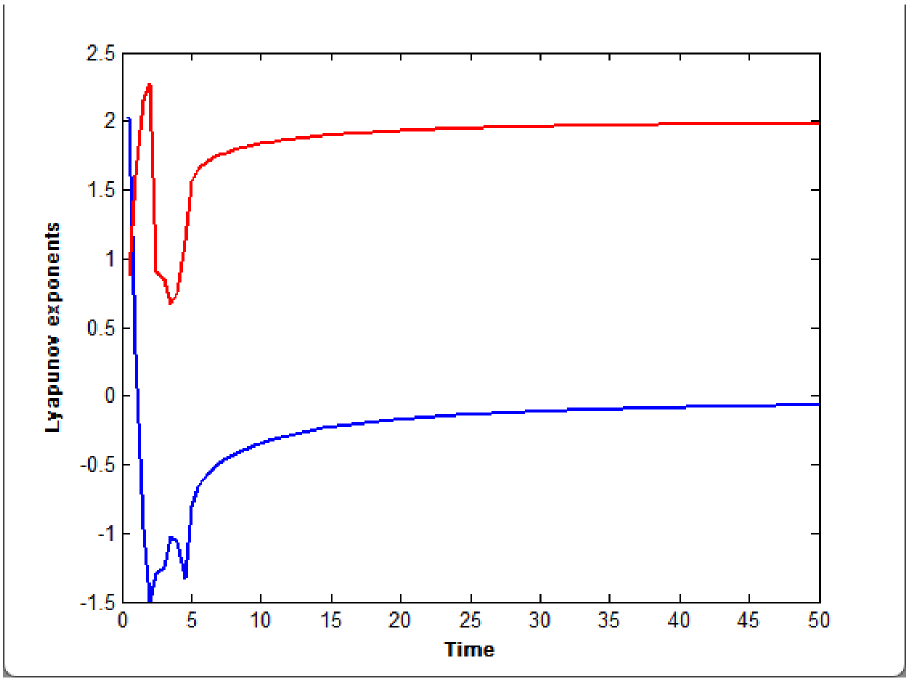


Figure 6. Lyapunov exponents for periodontium system (1)

CONTROLLING TECHNIQUES

In this section, we will use adaptive control method and active control method to control the chaotic periodontium system (1).

Adaptive control technique

Theoretical findings

For Stabilizing the chaotic behavior of periodontium system (1). We've designed an adaptive control law that incorporates the unknown parameter b

$$\left.\begin{aligned}\dot{G} &= 4g(1-g) - bgp + u_1 \\ \dot{P} &= 3.5\, gp + u_2\end{aligned}\right\} \dots\dots\dots\dots\dots\dots\dots..(8)$$

When u_1 *and* u_2 are the adaptive biological controllers, the adaptive control functions are:

$$\left.\begin{aligned}u_1 &= -4g(1-g) + \hat{b}gp - \mu_1 g \\ u_2 &= -3.5\, gp - \mu_2 p\end{aligned}\right\} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (9)$$

Where the constants μ_1 *and* μ_2 are greater than zero, $\hat{b}$ represents the parameter estimate of b. by substituting (9) into (8) we obtain

$$\left.\begin{aligned}\dot{G} &= -(b-\hat{b})gp - \mu_1 g \\ \dot{P} &= -\mu_2 p\end{aligned}\right\} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots...(10)$$

We define parameter estimation error by:

$$\mathrm{e}_b = b - \hat{b} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.\dots(11)$$

Using (11), the dynamics (10) can be written as

$$\left.\begin{aligned}\dot{G} &= -\,\mathrm{e}_b\, gp - \mu_1 g \\ \dot{P} &= -\mu_2 p\end{aligned}\right\} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots..\dots(12)$$

The Lyapunov approach is applied to derive the update law (12) which is used to adjust estimation of parameter b,

Count the Lyapunov Function

$$v(G,P) = \frac{1}{2}[g^2 + p^2 + \mathrm{e}_b^2] \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots...(13)$$

Which is positive definite function on $\mathbb{R}^3$

Also, Differentiating the parameter estimation errors (11) with respect to time, we get

$$\dot{\mathrm{e}}_b = -\dot{\hat{b}} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots...(14)$$

With above choices (13) and (14), the time derivation of the Lyapunov function along the trajectory has the following form:

$\dot{v}(G,P) = -\mu_1 g^2 - \mu_2 p^2 + e_b\left(-g^2 p - \dot{\hat{b}}\right)$ … … … … … … (15)

In equation (15), we update estimated parameter by:

$\dot{\hat{b}} = -g^2 p + \mu_3 e_b$ … … … … … … … … . . … … … … . (16)

Where the constant μ_3 is greater than zero.

Now, we substitute (16) in to (15), we get:

$\dot{v}(G,P) = -\mu_1 g^2 - \mu_2 p^2 - \mu_3 e_b^2$ … … … … … . . … (17)

Which is negative definite function on $\mathbb{R}^3$

The following result is so obtained by applying roots of characteristic equation, Routh stability criterion, Hurwitz stability criterion and Lyapunov stability.

Proposition 4: the chaotic periodontium system (8) is stabilized for three equilibrium points $E_0, E_1, E_2 \in \mathbb{R}^2$ by the adaptive biological control law (9) and the update estimated parameter law $\dot{\hat{b}} = -g^2 p + \mu_3 e_b$ where μ_1, μ_2 *and* μ_3 are positive constants and the results are shown in tables (4-7)

Numerical findings

The controlled system (12) is simulated with the following initial values:

$[G_0, P_0] = [0.00005, 0.00005]$ *and* $[\mu_1, \mu_2] = [10, 15]$. Figure7 shows controlled dynamics of Lyapunov exponents for periodontium system (1). Also, the trajectories of the controlled system (12) is simulated at initial condition $[G_0, P_0] = [0.004, 0.004]$. Figure8 shows the trajectories of the update system (12) after adaptive control, where (i) shows trajectories of state variables and (ii) shows Error trajectory goes to zero. From these trajectories it is easy to show how effectively the adaptive control law suppress the chaos in periodontium system (1).

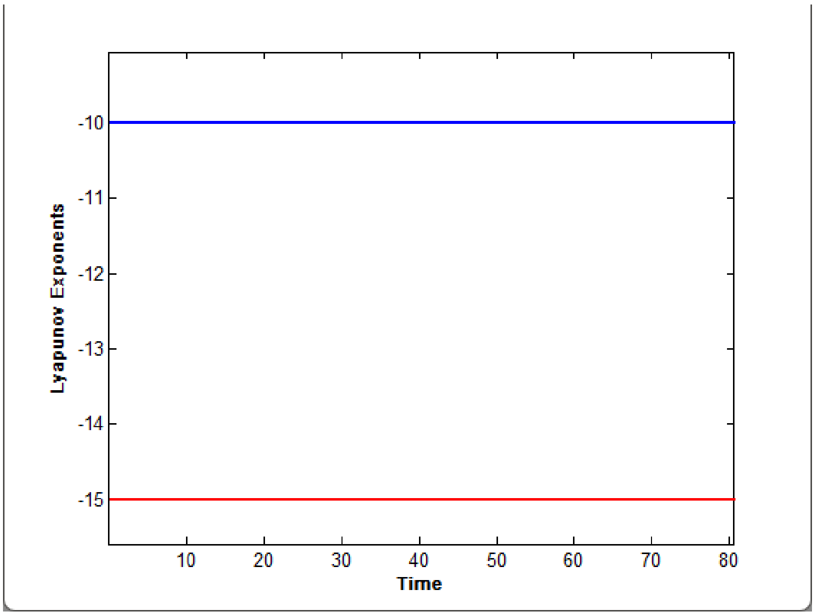
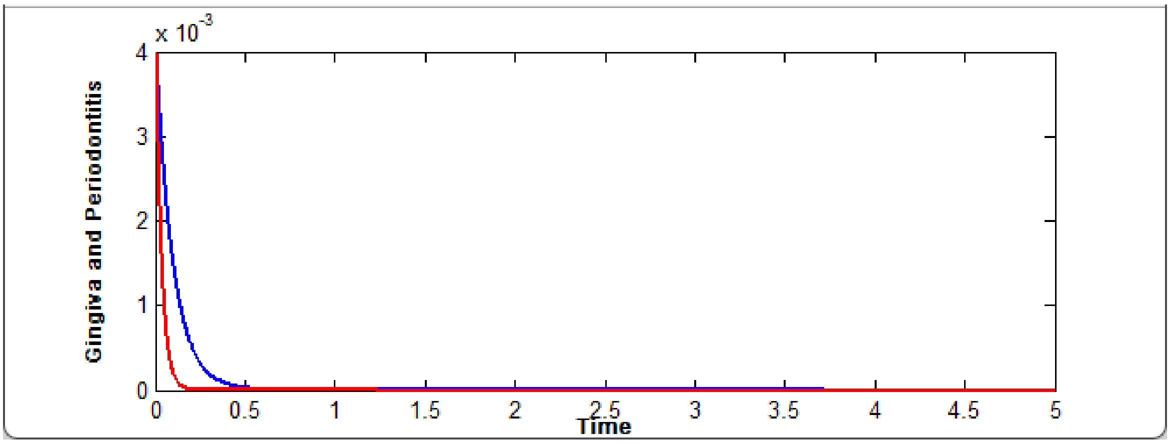


Figure 7. Dynamics of Lyapunov exponents after adaptive control



(i)

(ii)

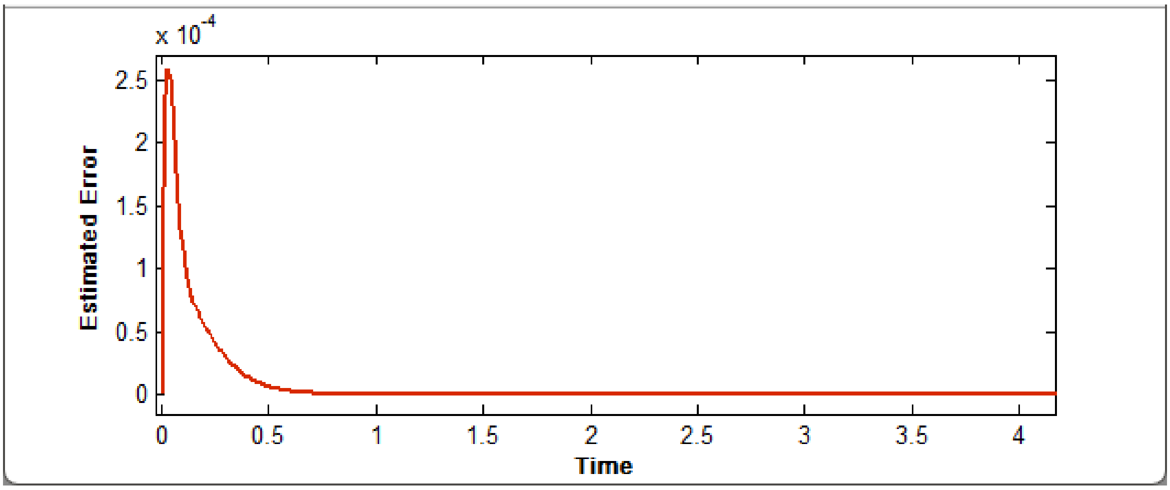


Figure 8. Trajectories of the update system (12) after adaptive control
(i) trajectories of state variables over time, (ii) trajectory of estimated error over time

Active control technique

Theoretical findings

For Stabilizing the chaotic behavior of periodontium system (1). We've designed an adaptive control law as follows:

$$\left.\begin{aligned} \dot{G} &= 4g(1-g) - 0.2gp + u_1 \\ \dot{P} &= 3.5gp + u_2 \end{aligned}\right\} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.(18)$$

Where u_1 *and* u_2 are the active feedback controllers, which are needed to be chosen such that the trajectory of periodontium system (1) is stabilized. The active control functions are:

$$\left.\begin{aligned} u_1 &= -4g(1-g) + 0.2gp - \mu_1 g \\ u_2 &= -3.5\,gp - \mu_2 p \end{aligned}\right\} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (19)$$

Where the constants μ_1 *and* μ_2 are positive, substituting (19) in to (18), we get:

$$\left.\begin{aligned} \dot{G} &= -\mu_1 g \\ \dot{P} &= -\mu_2 p \end{aligned}\right\} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.\dots\dots (20)$$

This shows that the dynamics (20) based on roots of characteristic equation, Routh stability criterion, Hurwitz stability criterion and Lyapunov stability gives the following result.

Proposition 5: the chaotic periodontium system (18) is stabilized for three equilibrium points $E_0, E_1, E_2 \in \mathbb{R}^2$ by the active nonlinear controllers (19) where μ_1 *and* μ_2 are positive constants and the results are shown in tables (4-7)

Numerical findings

The controlled system (20) is simulated with the following initial values:
$[G_0, P_0] = [0.00005, 0.00005]$ *and* $[\mu_1, \mu_2] = [10, 15]$. The controlled dynamics of Lyapunov exponents for periodontium system (1) are displayed in Figure9.

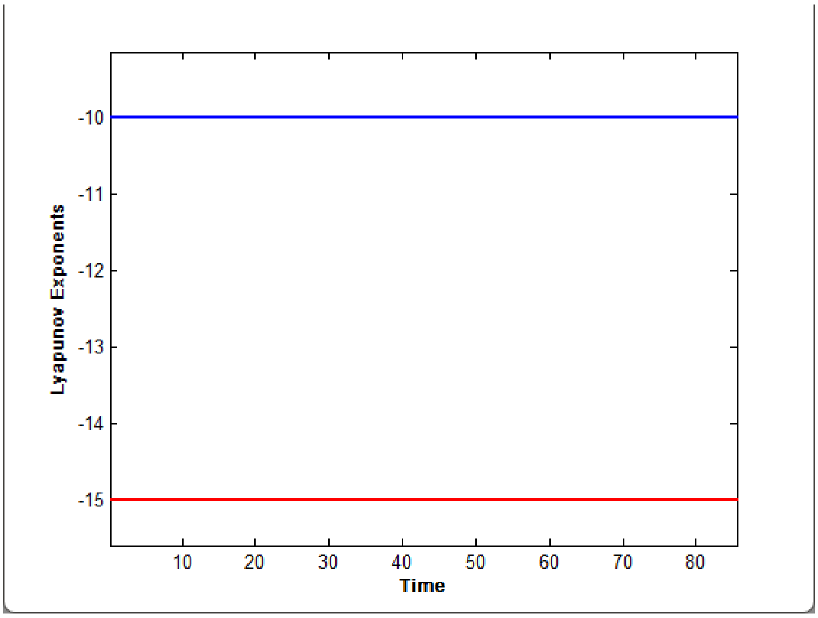


Figure 9. Dynamics of Lyapunov exponents after active control

According to the solutions and performance applied above, it is clear that the active control law is easier than the adaptive. Although the adaptive control is better due to the control parameter that has to be estimated. Lastly, both have controlled the system effectively.

SYSTEM COMPARISON TABLES BEFORE CONTROLLING TECHNIQUES AND AFTER

Table 4. Eigenvalues for periodontium system (1) before controlling techniques and after

Equilibrium points	Before control	After adaptive control	After active control
$E_0 = (0,0)$	$k_1 = 0$ $k_2 = 4$ unstable	$k_1 = -15$ $k_2 = -10$ stable	$k_1 = -15$ $k_2 = -10$ stable
$E_1 = (1,0)$	$k_1 = -4$ $k_2 = 3.5$ unstable	$k_1 = -15$ $k_2 = -10$ stable	$k_1 = -15$ $k_2 = -10$ stable
$E_2 = (0,P)$	$k_1 = 0$ $k_2 = 4 - 0.2p$ unstable	$k_{1,2} = -15$ stable	$k_1 = -15$ $k_2 = -10$ stable

Table 5. Routh-Array of periodontium system (1) before controlling techniques and after

Equilibrium points	k	Before control		k	After adaptive control		k	After active control	
$E_0 = (0,0)$	k^2 k^1 k^0	1 - 4 0	0 0 0	k^2 k^1 k^0	1 25 150	150 0 0	k^2 k^1 k^0	1 25 150	150 0 0
		unstable			stable			stable	
$E_1 = (1,0)$	k^2 k^1 k^0	1 0.5 -14	-14 0 0	k^2 k^1 k^0	1 25 150	150 0 0	k^2 k^1 k^0	1 25 150	150 0 0

		unstable			stable			stable	
$\boldsymbol{E_2 = (0, P)}$	k^2	1	0	k^2	1	$0.075p + 150$	k^2	1	150
	k^1	0.2p -4	0	k^1	$0.005p + 25$	0	k^1	25	0
	k^0	0	0	k^0	$0.075p + 150$	0	k^0	150	0
		unstable			stable			stable	

Table 6. Hurwitz criterion of periodontium system (1) before controlling techniques and after

Equilibrium points	Before control	After adaptive control	After active control
$\boldsymbol{E_0 = (0, 0)}$	$\Delta_1 = -4$ $\Delta_2 = 0$ unstable	$\Delta_1 = 25$ $\Delta_2 = 3750$ stable	$\Delta_1 = 25$ $\Delta_2 = 3750$ stable
$\boldsymbol{E_1 = (1, 0)}$	$\Delta_1 = 0.5$ $\Delta_2 = -7$ unstable	$\Delta_1 = 25$ $\Delta_2 = 3750$ stable	$\Delta_1 = 25$ $\Delta_2 = 3750$ stable
$\boldsymbol{E_2 = (0, P)}$	$\Delta_1 = 0.2p - 4$ $\Delta_2 = 0$ unstable	$\Delta_1 = 0.005p + 25$ $\Delta_2 = 0.000375p^2 + 2.625p + 3750$ stable	$\Delta_1 = 25$ $\Delta_2 = 3750$ stable

Table 7. Lyapunov Function of periodontium system (1) before controlling techniques and after

Equilibrium points	Before control	After adaptive control	After active control
$\boldsymbol{E_0 = (0, 0)}$	$v = 0$ $\dot{v} = 0$ unstable	$v = 0.0000125$ $\dot{v} = -0.0005$ stable	$v = 0$ $\dot{v} = 0$ stable
$\boldsymbol{E_1 = (1, 0)}$	$v = \frac{1}{2} g^2$ $\dot{v} = 0$ unstable	$v = 0.5000125$ $\dot{v} = -10.0005$ stable	$v = \frac{1}{2}$ $\dot{v} = -10$ stable
$\boldsymbol{E_2 = (0, P)}$	$v = \frac{1}{2} p^2$ $\dot{v} = 0$ unstable	$v = \frac{1}{2}[p^2 + 0.000025$ $\dot{v} = -15p^2 - 0.0005$ stable	$v = \frac{1}{2} p^2$ $\dot{v} = -15p^2$ stable

4. Conclusion

We believe that it is critical to preserve the dynamics in the favored degree so that the oral plants continue to be secure and stable for this purpose we tried to analyze the equilibrium points and the situations for the steadiness of the equilibria. In the periodontium model (1) observed three equilibrium factors, E_0=(0,0) is the extinction of populations. From organic factor of view it is superior periodontitis, the alveolar bone around the tooth is destroyed leading to enamel loosening and eventual tooth loss.$E_1 = (1,0)$ is the equilibrium with periodontitis extinct and the last equilibrium point $E_2 = (0, p)$ is the equilibrium with gingiva extinct which also means the advanced periodontitis. After studying this pathological case and concluding that the periodontium system (1) is in dissipative, bifurcation and unstable state, where the stability was tested using four methods: the roots of characteristic-equation, the Routh criterion, the Hurwitz criterion and Lyapunov Function. We proceeded to treat the periodontium system (1) using adaptive control technique and active control technique. The controlling was successful, We suppressed the periodontitis and effectively maintained the gingiva in wholesome kingdom. We have included comparison tables for all observe effects before and after control. Finally, using mathematical fashions in the subject of periodontology could permit the researchers to expect the pattern of hobby and the response circumstance in periodontium systems.

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