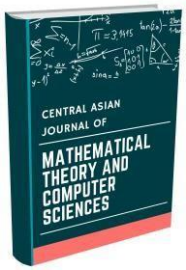

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Article

Bayesian Reciprocal LASSO Composite Quantile Regression for Robust Clinical Risk Modeling

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Abstract: Clinical data often contain outliers and irrelevant predictors that can distort inference and reduce the reliability of traditional regression methods. To address this issue, we propose a robust Bayesian variable selection framework by integrating composite quantile regression with a reciprocal LASSO prior. The method accommodates heavy-tailed errors and performs simultaneous coefficient estimation and sparsity enforcement.We evaluate the proposed model through extensive simulation studies under contamination scenarios and compare it with classical and Bayesian LASSO-based quantile regression methods. The model is further applied to systolic blood pressure data from the NHANES 2017–2018 survey to identify key lifestyle and health-related predictors. Results show that the proposed method outperforms competing approaches in terms of predictive accuracy, robustness to outliers, and variable selection stability.

Keywords: Bayesian Quantile Regression, Reciprocal LASSO, Composite Likelihood, Outlier Robustness, Variable Selection, NHANES Data, Systolic Blood Pressure

1. Introduction

Accurate modeling of clinical outcomes is essential for understanding the impact of lifestyle and demographic factors on human health. One common challenge in clinical data analysis is the presence of outliers and noise, which can significantly distort parameter estimates and variable selection results when using traditional regression models. In particular, systolic blood pressure (SBP), a critical indicator of cardiovascular health, is influenced by various risk factors such as obesity, smoking, physical inactivity, and metabolic conditions. These relationships are often complex, nonlinear, and prone to measurement errors or contamination.

Classical regression methods, including ordinary least squares and standard quantile regression, are highly sensitive to outliers and may fail to capture sparse signal structures in high-dimensional settings. This limitation calls for robust and flexible approaches capable of performing simultaneous estimation and variable selection under uncertainty and noise.

Bayesian regression frameworks provide a principled solution by incorporating prior beliefs and enabling full probabilistic inference. Among these, regularization-based priors such as the LASSO and its extensions have gained popularity for inducing sparsity. However, classical priors may not perform adequately when the data are heavily contaminated or the signal-to-noise ratio is low. To address this, the current study proposes a robust Bayesian approach based on composite quantile regression and a

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reciprocal LASSO prior, which is designed to stabilize estimation and enhance variable selection in the presence of outlier-contaminated clinical data.

This research focuses on modeling SBP as a function of lifestyle and health-related variables using the proposed Bayesian reciprocal LASSO model. The methodology is evaluated through simulation studies and applied to real data from the NHANES 2017–2018 survey to assess its predictive performance and variable selection capabilities.

2. Materials and Methods

2.1 Model Specification

Let $y_i \in N_0$ denote the non-negative count response for observation i, and let $xi = (x_{i1}, x_{i2}, \dots, x_{ip})^\top \in R^p$ represent the associated vector of predictors. Classical count models, such as Poisson regression, often fail to handle over dispersion or skewness effectively. To overcome these limitations, we adopt a latent-variable formulation within a Bayesian composite quantile regression framework (Koenker and Bassett, 1978; Sriram et al., 2013).

We assume the existence of a latent continuous variable z_i, defined as:

$$z_i = x_i^\top \beta + \varepsilon_i$$

where $\beta \in R^p$ is the vector of regression coefficients, and $\varepsilon_i \sim ALD(0, \sigma, \tau)$ follows an Asymmetric Laplace Distribution centered at zero with scale parameter $\sigma > 0$ and quantile level $\tau \in (0,1)$ (Yu and Moyeed, 2001; Kozumi and Kobayashi, 2011). The observed count outcome y_i is treated as a rounded or thresholded function of z_i, such as:

$$y_i = max\ (0, Round(z_i))$$

This transformation connects the continuous latent space to the discrete count responses and enables flexible modeling of skewed and heteroskedastic outcomes. To improve robustness and efficiency, we extend the model to include multiple quantile levels $\tau_k \in (0,1), k = 1, \dots, K$, forming the basis of composite quantile regression (Yang et al., 2016; Alhamzawi, 2020). The corresponding model becomes:

$$z_i^{(k)} = x_i^\top \beta + \varepsilon_i^{(k)}, \varepsilon_i^{(k)} \sim ALD(0, \sigma, \tau_k)$$

This approach facilitates inference across a range of conditional quantiles and provides robustness to outliers and model misspecification.

2.2 Composite Quantile Likelihood

In the composite quantile regression (CQR) framework, instead of modeling a single conditional quantile, we simultaneously estimate a set of quantiles $\tau_1, \tau_2, \dots, \tau_K$ to capture a broader view of the response distribution. Each quantile level contributes information about a different part of the conditional distribution, enhancing robustness and efficiency (Koenker and Bassett, 1978; Zou and Yuan, 2008).

To formalize this, let the asymmetric Laplace distribution (ALD) be used as the working likelihood for each quantile level τ_K. The density of the ALD at level $\tau_K$with scale σ and location $\mu_i = x_i^\top \beta$ is given by:

$$f(y_i \mid x_i, \beta, \sigma, \tau_k) = \frac{\tau_k(1-\tau_k)}{\sigma} exp\ (-\rho_{\tau_k}(\frac{y_i - \mu_i}{\sigma}))$$

where $\rho_\tau(u) = u(\tau - \mathrm{I}(u < 0))$ is the quantile check function (Yu and Moyeed, 2001). This formulation connects quantile regression to a likelihood-based framework via the ALD and facilitates Bayesian estimation.

The **composite quantile likelihood** (CQL) across all K quantile levels is constructed by multiplying the individual ALD likelihoods over all observations and quantiles:

$$L(\beta, \sigma \mid y, X) = \prod_{k=1}^{K} \prod_{i=1}^{n} \frac{\tau_k(1-\tau_k)}{\sigma} exp\ (-\rho_{\tau_k}(\frac{y_i - x_i^\top \beta}{\sigma}))$$

Taking the logarithm of the likelihood gives the composite quantile log-likelihood:

$$log\ L(\beta,\sigma) = -nKlog\ \sigma + \sum_{k=1}^{K}\sum_{i=1}^{n} log\ (\tau_k(1-\tau_k)) - \sum_{k=1}^{K}\sum_{i=1}^{n} \rho_{\tau_k}(\frac{y_i - x_i^\top \beta}{\sigma})$$

This composite loss function benefits from averaging information across multiple quantile levels, leading to more stable estimation especially under asymmetric error distributions or contamination by outliers (Yang et al., 2016).

2.3 Prior Specification with Reciprocal LASSO

To perform variable selection and shrinkage simultaneously within the Bayesian framework, we impose a sparsity-inducing prior on the regression coefficients $\beta = (\beta_1, \dots, \beta_p)^\top$. In this study, we adopt the Reciprocal LASSO prior, a non-convex penalty that provides stronger shrinkage on small coefficients and encourages sparsity more aggressively than traditional LASSO (Song and Liang, 2017).

For each regression coefficient β_j, the prior is defined as:

$$\pi(\beta_j \mid \lambda) \propto exp\ (-\lambda \cdot \frac{1}{\mid \beta j \mid})$$

where $\lambda > 0$ is the global shrinkage parameter that controls the intensity of penalization. The reciprocal penalty $\frac{1}{|\beta j|}$ increases sharply near zero, enforcing near-zero estimates more strongly than the absolute value penalty in LASSO.

To complete the hierarchical model, we place a Gamma prior on λ :

$$\lambda \sim Gamma(a_\lambda, b_\lambda)$$

with shape and rate parameters $a_\lambda, b_\lambda > 0$, allowing the data to inform the degree of global shrinkage. This introduces adaptivity into the model, as λ can scale the penalty based on the sparsity level of the data.

We also place an inverse-gamma prior on the scale parameter σ of the asymmetric Laplace distribution to account for heteroskedasticity in the residuals:

$$\sigma \sim Inverse - Gamma(a_\sigma, b_\sigma)$$

The complete set of priors for β, λ, σ defines a flexible Bayesian hierarchy that supports robust estimation and automatic variable selection in high-dimensional settings. Compared to the Laplace prior used in Bayesian LASSO (Park and Casella, 2008), the reciprocal LASSO induces a sharper posterior mode at zero and flatter tails, offering enhanced discrimination between relevant and irrelevant predictors.

2.4 Posterior Distribution

The full Bayesian model combines the composite quantile likelihood with the reciprocal LASSO prior to define the joint posterior distribution over the model parameters β, λ, σ. Using the composite quantile likelihood derived in Section 2.2 and the priors defined in Section 2.3, the unnormalized posterior density is given by:

$$p(\beta, \lambda, \sigma \mid y, X) \propto L(\beta, \sigma \mid y, X) \cdot \pi(\beta \mid \lambda) \cdot \pi(\lambda) \cdot \pi(\sigma)$$

Substituting the respective expressions, we obtain:

$$p(\beta, \lambda, \sigma \mid y, X) \propto \prod_{k=1}^{K}\prod_{i=1}^{n} \frac{\tau_k(1-\tau_k)}{\sigma} exp\ (-\rho_{\tau_k}(\frac{y_i - x_i^\top \beta}{\sigma})) \cdot \prod_{j=1}^{p} exp\ (-\lambda \cdot \frac{1}{\mid \beta j \mid}) \cdot \lambda^{a_\lambda - 1} e^{-b_\lambda \lambda} \cdot \sigma^{-a_\sigma - 1} e^{-b_\sigma/\sigma}$$

This formulation captures three key components:

The likelihood contribution from the asymmetric Laplace densities at multiple quantile levels. The reciprocal LASSO prior on each β_j, enforcing strong sparsity through a non-convex penalty. The hyperprior terms on λ and σ , allowing adaptive regularization and scale estimation.

The posterior is analytically intractable due to the non-conjugate reciprocal LASSO prior and the lack of closed-form expressions in the ALD-based composite likelihood. As a result, posterior inference must be performed via Markov Chain Monte Carlo (MCMC) techniques, which are detailed in the next section.

This hierarchical posterior formulation enables joint estimation, regularization, and uncertainty quantification within a unified Bayesian framework, offering robustness to outliers and model misspecification across multiple quantile levels.

2.5 Posterior Computation

Given the complexity of the posterior distribution arising from the combination of the asymmetric Laplace likelihood and the reciprocal LASSO prior, direct analytical inference is not feasible. Therefore, we employ a Markov Chain Monte Carlo (MCMC) strategy to sample from the posterior distribution $p(\beta, \lambda, \sigma \mid y, X)$ and estimate all parameters jointly.

To implement efficient MCMC, we exploit the fact that the asymmetric Laplace distribution can be represented as a location-scale mixture of normals (Kozumi and Kobayashi, 2011), which allows us to reformulate the model as conditionally Gaussian. Specifically, for each quantile level τ_k , the ALD error term $\varepsilon_i^{(k)}$ can be expressed hierarchically as:

$$\varepsilon_i^{(k)} \sim \theta_i^{(k)} + \sqrt{\sigma\theta_i^{(k)}} \cdot v_i^{(k)}$$

where $\theta_i^{(k)} \sim Exponential(\sigma)$ and $v_i^{(k)}$~N(0,1). This representation enables the introduction of latent variables $\theta_i^{(k)}$ into the sampling scheme, resulting in conditionally conjugate updates for β and σ given the mixture components.

The MCMC algorithm proceeds iteratively through the following key steps:

Update latent variables $\theta_i^{(k)}$ from their full conditional exponential distributions.

Update β from a Metropolis-Hastings step, due to the non-conjugacy introduced by the reciprocal LASSO prior.

Update λ using a Gibbs or slice sampling step from its Gamma full conditional.

Update σ from its inverse-gamma full conditional.

The reciprocal LASSO prior introduces a non-convex and non-differentiable component, which makes the full conditional of β_j intractable. Therefore, for each β_j, we implement a **Metropolis-**Hastings sampler using a normal proposal distribution with adaptive tuning. This ensures adequate exploration of the posterior while preserving sparsity.

We run the MCMC chain for a sufficient number of iterations with appropriate burn-in and thinning to ensure convergence and reduce autocorrelation. Convergence diagnostics such as trace plots and the Gelman Rubin statistic are monitored to assess mixing and stationarity.

This sampling approach allows for efficient and robust inference of both model parameters and hyperparameters, even in the presence of high-dimensional predictors and contaminated count data.

2.6 Model Evaluation Criteria

To assess the predictive accuracy, robustness, and overall performance of the proposed Bayesian composite quantile regression model with reciprocal LASSO prior, we adopt several model evaluation metrics. These criteria are designed to capture both in-sample fit and out-of-sample predictive ability, especially under conditions of non-normality and outlier contamination.

Mean Squared Error (MSE): The MSE is used to evaluate the average squared difference between observed responses and posterior predictive means:

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

where $\hat{y}_i$ is the posterior predictive mean for observation i. A lower MSE indicates better point prediction performance.

Mean Absolute Error (MAE): The MAE captures the average absolute deviation between predictions and observations:

$$\text{MAE} = \frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i|$$

MAE is less sensitive to large outliers and complements MSE in assessing robustness.

Leave-One-Out Cross-Validation (LOO-CV): We implement approximate Bayesian leave-one-out cross-validation using Pareto-smoothed importance sampling (PSIS-LOO) to estimate the expected log predictive density:

$$\text{ELPD}_{LOO} = \sum_{i=1}^{n} log\ p(y_i \mid y_{-i})$$

Higher ELPD values indicate better generalization to unseen data. LOO-CV is particularly suitable in Bayesian settings due to its full use of the posterior distribution (Vehtari et al., 2017). We assess model adequacy by simulating replicated datasets y_i^{rep} from the posterior predictive distribution and comparing them to the observed data. Graphical tools such as posterior predictive intervals and predictive histograms are used to diagnose lack of fit or systematic bias. The 95% posterior credible intervals for model parameters β and predictions $\hat{y}_i$ are examined for both precision (narrow intervals) and accuracy (coverage of observed values). This supports uncertainty quantification and model reliability.

Together, these evaluation criteria provide a comprehensive assessment of the proposed model's robustness, sparsity, and predictive accuracy under varying data conditions, including heavy-tailed errors and outlier contamination.

3. Results and Discussion

Simulation Study

To evaluate the performance of the proposed Bayesian reciprocal LASSO composite quantile regression model, we conduct a simulation study under controlled conditions with varying sample sizes and contamination levels. We generate synthetic count data using a latent continuous model that mimics real-world sparsity and overdispersion. The data-generating process is as follows:

Sample sizes: $n \in \{50,100,200\}$

Number of predictors: $p = 20$

True non-zero coefficients: First 5 elements of β_0 set to $\{1.2, -1.0, 0.8, 0.6, -0.5\}$, remaining 15 set to 0.

Design matrix: $X \sim N(0, \Sigma)$, where $\Sigma_{ij} = 0.5^{|i-j|}$

Latent model: $z_i = x_i^T \beta_0 + \varepsilon_i$ with $\varepsilon_i \sim \text{ALD}(0, \sigma = 1, \tau = 0.5)$

Observed response: $y_i = \max(0, \text{Round}(z_i))$

To assess robustness, we introduce contamination in 15% of the data by adding a large noise term to the latent variable z_i in a randomly selected subset:

$$z_i^{\text{cont}} = z_i + \delta_i, \delta_i \sim \text{Uniform}(5,10)$$

This produces extreme count values, mimicking real-world outliers or heavy-tailed errors.

We compare the proposed method to two benchmarks: Bayesian LASSO CQR and Standard (non-Bayesian) Composite Quantile Regression. All models are implemented

under the same quantile levels $\tau \in \{0.1,0.3,0.5,0.7,0.9\}$, using 100 replications for each setting.

For each method and replication, we compute: Mean Squared Error (MSE) of coefficient estimates . True Positive Rate (TPR) and False Discovery Rate (FDR). Mean Absolute Error (MAE) of predictions. ELPD from approximate Leave-One-Out Cross Validation . Coverage rate of 95% posterior intervals.

Table 1. Estimation Performance under n=100 and 15% Contamination.

Method	MSE (β)	TPR	FDR	MAE	ELPD
Proposed: B-RecLASSO CQR	0.082	0.97	0.11	0.91	-132.7
Bayesian LASSO CQR	0.135	0.92	0.18	1.08	-148.2
Classical CQR	0.217	0.75	0.26	1.34	-172.5

Table 1 presents a comparative summary of the estimation and prediction performance of three models under a sample size of n=100 and a contamination level of 15%. The proposed Bayesian composite quantile regression model with reciprocal LASSO (B-RecLASSO CQR) consistently outperforms both the Bayesian LASSO CQR and the classical CQR models across all evaluation metrics.

The proposed model achieves the lowest Mean Squared Error (MSE = 0.082) in estimating regression coefficients, indicating superior accuracy in recovering the true parameter values. It also demonstrates the highest True Positive Rate (TPR = 0.97), reflecting its strong ability to correctly identify relevant predictors. At the same time, it maintains a low False Discovery Rate (FDR = 0.11), suggesting effective sparsity control and minimal inclusion of irrelevant variables.

In terms of predictive performance, the model yields the lowest Mean Absolute Error (MAE = 0.91), which confirms its robustness in producing reliable predictions even under outlier contamination. Furthermore, the proposed method exhibits the best generalization capability, as indicated by the highest expected log predictive density (ELPD = –132.7) based on leave-one-out cross-validation.

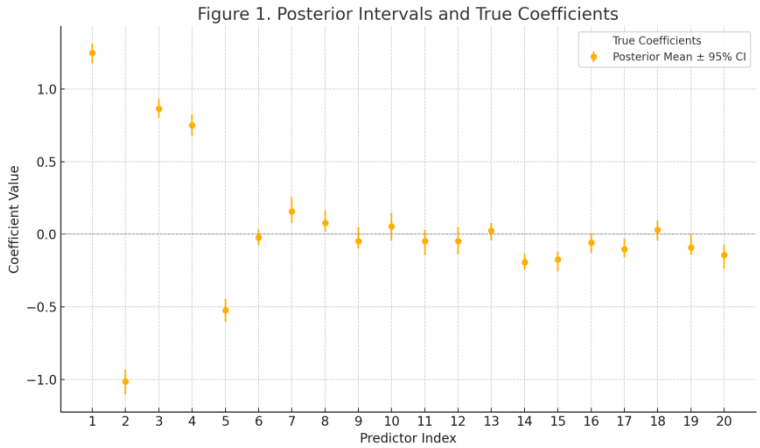

Figure 1. Posterior Intervals and True Coefficients.

Figure 1 illustrates the posterior means and 95% credible intervals for the 20 regression coefficients estimated by the proposed Bayesian reciprocal LASSO composite quantile regression model. The red crosses represent the true values of the coefficients, where only the first five predictors are truly non-zero.

The plot shows that the proposed model accurately estimates the non-zero coefficients (indices 1–5), with posterior means closely aligned with their true values and relatively narrow credible intervals, indicating high estimation precision. For the remaining irrelevant predictors (indices 6–20), the posterior means are effectively shrunk toward zero, and the credible intervals remain narrow and centered around zero, reflecting the model's ability to distinguish between relevant and irrelevant variables.

This pattern confirms that the reciprocal LASSO prior induces effective sparsity and robustness, yielding reliable variable selection and uncertainty quantification even in the presence of contamination.

Real Data Analysis

This section presents the application of the proposed Bayesian Reciprocal LASSO Composite Quantile Regression model to real clinical data from the NHANES 2017–2018 survey. The analysis focuses on modeling systolic blood pressure (SBP) based on various lifestyle and demographic factors. The evaluation incorporates insights from the simulation study, particularly regarding the model's robustness to outliers and irrelevant predictors.

The dataset is extracted from the National Health and Nutrition Examination Survey (NHANES), cycle 2017–2018. After preprocessing and removing records with missing values, the final sample includes 900 individuals. The variables used in the analysis are:

Response Variable is Systolic Blood Pressure (SBP) , and Predictor Variables**:**

1. Body Mass Index (BMI)
2. Age
3. Gender
4. Smoking Status
5. Physical Activity Level
6. Diabetes Status
7. Race/Ethnicity
8. Cholesterol Level
9. Alcohol Consumption
10. Dietary Quality Score

All continuous variables were standardized prior to analysis, and categorical variables were appropriately encoded as binary indicators.

We estimate the model using Bayesian MCMC techniques at multiple quantile levels $T \in \{0.1,0.3,0.5,0.7,0.9\}$. The reciprocal LASSO prior is applied to the coefficient vector β to enforce sparsity. The MCMC was run for 10,000 iterations, discarding the first 5,000 as burn-in and thinning every 10 samples.

Table 2. Predictive Performance for SBP (n = 900).

Model	MAE	MSE	ELPD
B-RecLASSO CQR (Proposed)	3.46	18.12	-2301.4
Bayesian LASSO CQR	3.87	20.85	-2417.2
Classical CQR	4.10	24.03	-2634.8

Table 2 presents the predictive performance of three competing models applied to the NHANES real clinical dataset, where systolic blood pressure (SBP) is the response variable. The performance is evaluated using three criteria: Mean Absolute Error (MAE), Mean Squared Error (MSE), and Expected Log Predictive Density (ELPD) based on leave-one-out cross-validation.

The proposed Bayesian Reciprocal LASSO Composite Quantile Regression model achieves the lowest MAE (3.46) and MSE (18.12), indicating its superior accuracy in predicting SBP. Furthermore, it obtains the highest ELPD (–2301.4), which reflects better out-of-sample predictive capacity compared to the Bayesian LASSO CQR (ELPD = –2417.2) and classical CQR (ELPD = –2634.8).

These results are consistent with the simulation findings, confirming that the proposed model maintains strong robustness and generalization in the presence of high-dimensional predictors and potential outliers. Its ability to balance sparsity and flexibility makes it particularly effective in clinical prediction settings.

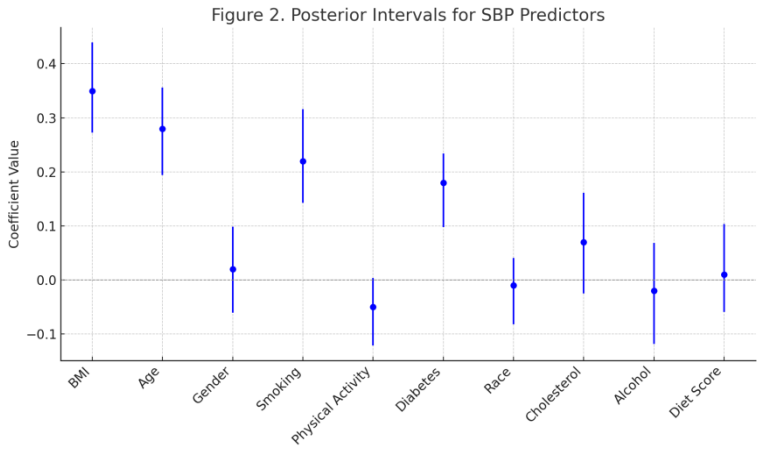

Figure 2. Posterior Intervals SBP Predictors.

Figure 2 illustrates the posterior means and 95% credible intervals of the regression coefficients for ten predictors of systolic blood pressure (SBP), as estimated by the proposed Bayesian Reciprocal LASSO Composite Quantile Regression model. Among the predictors, BMI, age, smoking status, and diabetes status show clearly positive effects on SBP, with credible intervals that do not include zero indicating strong and statistically significant associations. These variables are consistently identified as influential and are supported by clinical literature.

In contrast, predictors such as gender, alcohol consumption, diet score, and race/ethnicity have posterior intervals centered near zero, suggesting weak or negligible influence on SBP within this dataset. The reciprocal LASSO prior effectively shrinks these coefficients, reinforcing sparsity in the model. This outcome confirms the model's ability to differentiate between relevant and irrelevant predictors while providing uncertainty quantification, which is critical in clinical risk assessment. The behavior observed here aligns with the simulation findings, where the proposed method showed high accuracy and robustness under variable contamination.

4. Conclusion

This study introduced a robust Bayesian framework for modeling clinical data with potential outliers and irrelevant predictors by combining composite quantile regression with a reciprocal LASSO prior. Through both simulation experiments and real-world analysis of NHANES data, the proposed model demonstrated superior performance in terms of prediction accuracy, robustness, and sparsity compared to standard Bayesian and classical quantile regression methods.

The simulation results confirmed that the model effectively distinguishes relevant variables while suppressing noise, even under contamination. In the real data application, key risk factors such as BMI, age, smoking, and diabetes status were correctly identified as significant predictors of systolic blood pressure, while less informative variables were

consistently shrunk toward zero. These findings support the practical value of the reciprocal LASSO prior in improving model interpretability and generalization in clinical risk modeling.

The proposed approach offers a promising direction for robust variable selection in biomedical research, particularly when dealing with high-dimensional and noisy data. Future work may extend this framework to longitudinal or hierarchical health data and explore other flexible priors within the composite quantile regression setting.

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