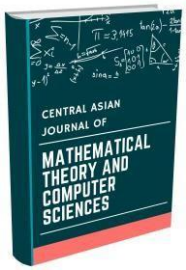

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Article

Analysis of Probability and Statistics Topics in Mathematics Textbooks (Singapore, Japan, USA, Russia)

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Abstract: Teaching the subject of mathematics has always been considered relevant in every era. Mastering this subject well not only affects students' knowledge but also has a significant impact on indicators that determine their future. Especially, the knowledge related to the sections of Probability Theory and Statistics in mathematics is being used in many fields. For example, constructing models of research conducted in all disciplines, evaluating them, showing their validity, and making forecasts all require knowledge from this subject. In addition, Probability Theory and Statistics help develop logical reasoning, gain profits in business through risk-taking, and choose the option with the highest probability among different life choices. For instance, the world's major investors rely on probability and statistical data when investing in a field, country, or entrepreneur that promises good returns. This article analyzes the teaching of Probability Theory and Mathematical Statistics sections in general education school textbooks through the experiences of foreign countries (Singapore, Japan, USA, Russia). That is, according to the curriculum, the topics of Probability Theory and Mathematical Statistics are given in mathematics textbooks for grades 7–11, and the composition, sequencing, and interconnection with previous and subsequent topics are explained. The diversity and real-life relevance of the problems presented in textbooks are also discussed.

Keywords: Probability theory, statistics, textbook, curriculum, random event, probability, statistical characteristics

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1. Introduction

Teaching mathematics well and being able to interest students in this subject requires mathematics teachers to constantly work on improving themselves. Moreover, the mathematics textbook also plays a major role in this process [1]. The mathematics textbook organizes problems and concepts related to mathematics and serves as a central point in the process of teaching mathematics [2]. The textbook acts as a part of the didactic process jointly organized by the mathematics textbook, teacher, and student [3]. Analyzing textbooks is useful for understanding what is required when developing teaching and curriculum approaches, and for identifying appropriate methods. In empirical research conducted up to 2025, central questions in textbook analysis included which types of learning are effective, which topics and grades should be emphasized [4]. Three analytical foundations have been proposed for analyzing problems in textbooks: modes of expression, context characteristics, and types of answers [5]. These elements facilitate the analysis of problems presented in textbooks. It is necessary to choose a textbook that is appropriate for students – that is, one that is easy to understand, places more emphasis on practical application rather than theory, and contains various and comprehensive problems related to the topics [6], [7]. In other words, in textbook analysis, these aspects are considered important, and the topics in textbooks are required to be based on didactic knowledge. Because for teachers as well, the most correct and effective teaching method

is said to be "Didactic knowledge" [8]. Currently, when analyzing algebra textbooks, we encounter several shortcomings such as lack of consistency in the sequencing of topics and concepts related to *Probability Theory* and *Mathematical Statistics*, lack of developing and real-life problems, and limitation to standard problems [9], [10]. Indeed, these shortcomings reduce the level of mastery of the subject. However, in recent years, with the emergence of interactive educational technologies and new methodological approaches, significant innovations have been observed in the teaching of *Probability Theory* and *Mathematical Statistics*. Thus, the analysis of mathematics textbooks helps solve many problems in the learning process and knowledge acquisition. In particular, it is expected that analyzing the topics and problems related to *Probability Theory* and *Statistics* included in the mathematics textbooks of Singapore, Japan, the United States, and Russia will contribute to improving and increasing students' mastery of these topics. This expectation is grounded in the fact that *Probability Theory* and *Mathematical Statistics* are highly useful in daily life, are essential for many professions due to the need for stochastic knowledge, and play a significant role in developing critical thinking skills [11].

The main goal of this research is to find answers to the following questions:

1. Which topics in the *Probability Theory* and *Statistics* sections are included in the mathematics textbooks of Singapore, Japan, the United States, and Russia?
2. With which topics are the contents and problems of *Probability Theory* and *Statistics* linked in the mathematics textbooks of these countries?
3. Which topics in the *Probability Theory* and *Statistics* sections are appropriate to be included in the mathematics textbooks of general education schools?

2. Materials and Methods

This study employs textbook collection, analysis, and comparison methods. The purpose of these methods is to analyze, compare, evaluate the collected data, and draw conclusions. The research covers the mathematics textbooks of Singapore, Japan, the United States, and Russia. For Singapore, textbooks from grades 7 to 10 were used, while for Japan, the United States, and Russia, textbooks from grades 7 to 11 were examined. The topics in the Singaporean textbooks were taken from *The Singapore New Syllabus Mathematics (NSM)* grades 7–10, written by Dr. Joseph Yeo PhD, MEd, PGDE (Dist.), BSc (Hons), Teh Keng Seng BSc, Dip Ed, Loh Cheng Yee BSc, Dip Ed, and Ivy Chow MEd, PGDE, BSc. These textbooks were developed based on the curriculum guidelines of the Singapore Ministry of Education. The *New Syllabus Mathematics* (NSM) series had the highest market share in Singapore, accounting for approximately 80% of mathematics textbooks. The mathematics curricula of Japan were sourced from: https://www.futureschool.com/japan-curriculum/#552f669b605a0

The standards for the subject of mathematics in the state of California, USA, were analyzed based on data from: https://www.ixl.com/standards/california/math

The Russian mathematics curriculum was based on the following textbooks:

1. Макарычев Ю.Н., Миндюк Н.Г., Нешков К.И. et al., *Algebra* Grade 7 and Grade 9,
2. Рубин А.Г., Чулков П.В., *Algebra* Grade 8,
3. Муравин Г.К., Муравина О.В., *Algebra and the Fundamentals of Mathematical Analysis* Grade 10,
4. Мерзляк А.Г., Номировский Д.А., Поляков В.М., *Algebra and the Fundamentals of Mathematical Analysis* Grade 11.

In this study, the topics related to *Probability Theory* and *Mathematical Statistics* presented in mathematics textbooks from grades 7 to 11 were analyzed. Specifically, the research examined:

1. In which grades are topics on probability and statistics introduced,
2. In which sequence and with which other topics these concepts are taught,
3. Which topics should be incorporated into the curriculum to effectively teach probability and statistics in general education schools.

Mathematics textbooks play a critical role in improving the quality of the educational process. Therefore, research focused on textbook analysis has been increasing [12], [13]. Indeed, textbooks have a strong influence on teaching and learning processes. In particular, the distribution of topics on probability and statistics across grades significantly impacts students' understanding of the subject.

In this research, the mathematics textbooks and curricula of general education schools (grades 7–11) in Singapore, Japan, the United States, and Russia were analyzed. Topics from probability such as random events, calculating event probabilities, and elements of combinatorics, and from statistics such as statistical characteristics, polygons, histograms, etc., were examined. The inclusion and sequence of these topics vary by country.

3. Results and Discussion

Results.

In the Table 1. Shows that The distribution of *Probability Theory* and *Mathematical Statistics* topics in mathematics textbooks shows that in Singaporean textbooks, a total of 18 topics are allocated to this section: 11 related to probability and 7 to statistics. In Japan, there are 36 topics, of which 20 are dedicated to probability and 16 to statistics. In the United States, 40 topics were identified: 27 on probability and 13 on statistics. In Russia, 26 topics were assigned: 22 for probability and 4 for statistics. It is observed that *Probability Theory* topics are more prevalent than statistics in the mathematics textbooks of Singapore, the USA, and Russia, whereas Japan allocates more hours to statistics topics.

Table 1. Topics on Probability Theory and Mathematical Statistics in Mathematics Textbooks (Grades 7–11)

Grade / Country	SINGAPORE	JAPAN	USA (California)	RUSSIA
Grade 7	Statistical Data Processing	Data Handling	Identify representative, random, and biased samples	Статистическая характеристика (Statistical Characteristics)
	Bar Charts, Pie Charts, Line Graphs	Probability of simple and compound events	Make predictions using theoretical and experimental probability	
			Probability of independent and dependent events	
Grade 8	Introduction to Probability	Frequency distribution tables	Create scatter plots	Статистические характеристики (Statistical Characteristics)
	Sample Space	Frequency histograms and polygons	Identify trends with scatter plots	Таблицы частот (Frequency Tables)
	Probability of Single Events	Relative frequency	Outliers in scatter plots	Интервальный метод (Interval Method Concept)
	Simple Combined Events	The Range	Identify lines of best fit	

Grade / Country	SINGAPORE	JAPAN	USA (California)	RUSSIA
	Possibility Diagrams and Tree Diagrams	The Mode	Write equations for lines of best fit	
		The Mean	Interpret lines of best fit	
Grade 9	Collect, classify and tabulate statistical data	Frequency distribution tables	Interpreting categorical and quantitative data	Высказывания и предикаты. Кванторы (Statements and Quantifiers)
	Statistical Diagrams	Frequency histograms and polygons	Create histograms	Операции над высказываниями и предикатами (Operations on Statements)
	Polygons, Histograms	Relative frequency	Mean, median, mode, and range	Конъюнкция, дизъюнкция, импликация (Conjunction, Disjunction, Implication)
	Mean, Median, Mode for individual data	The Range	Variance and Standard Deviation	Перестановки, размещения, сочетания (Permutations, Arrangements, Combinations)
	Mean, Median, Mode for grouped data	The Mode	Two-way frequency tables	Частота и вероятность (Frequency and Probability)
		The Mean	Regression line analysis	Сложение и умножение вероятностей (Addition and Multiplication of Prob.)
		The Median	Correlation coefficient	Испытания Бернулли (Bernoulli Trials)
				Числовые характеристики распределения вероятностей (Distribution Stats)

Grade / Country	SINGAPORE	JAPAN	USA (California)	RUSSIA
Grade 10	Statistical Data Analysis	Frequency distribution tables	Theoretical and experimental probability	Понятие о вероятности (Concept of Probability)
	Single Event Probability Calculation	Cumulative Frequency	Independent and Dependent Events Identification	Вычисление числа вариантов (Counting Principles)
	Combined Event Probability	Calculating the median from a frequency distribution	Conditional probabilities	
	Possibility and Tree Diagrams	Tree diagrams (no prior outcome dependency)	Two-way frequency tables	
		Tree diagrams (with prior outcome dependency)		
Grade 11	-	Probability of simple events	Normal distribution probability	Элементы комбинаторики и бином Ньютона (Combinatorics, Binomial Thm)
		Rolling a pair of dice	Identifying biased samples	Аксиомы теории вероятностей (Probability Axioms), Conditional Probability
		Experimental probability	Experimental design	Независимые события. Случайная величина (Indep. Events, Random Variables)
		The Complementary Result	-	Схема Бернулли. Биномиальное распределение (Bernoulli, Binomial Distr.)
		-	-	Характеристики случайной величины (Random Variable Characteristics)

Discussion

A. Singapore

According to the results of international assessments such as TIMSS and PISA, Singapore ranks among the top five countries [14]. In particular, based on the results of the **PISA 2022** assessment, Singapore ranked **first** out of 91 countries. This demonstrates the high quality and systematic organization of the country's educational system. In Singaporean mathematics textbooks, statistical topics begin in **Grade 7** with an introduction to statistical data. In **Grade 8**, topics such as *Introduction to Probability* and *Probability of Events* are introduced. In **Grade 9**, students are taught statistical topics such as *Collect, classify, and tabulate statistical data*, and *Calculate the mean, median, and mode for individual data*, as shown in Table 1. Prior to these topics, the textbooks cover functions and their graphs, which are helpful for constructing polygons and histograms. In **Grade 10**, the focus shifts to the analysis of statistical data and calculating probabilities of events.

B. Japan

According to Japanese mathematics teachers, mathematics education in Japan has significantly changed over the last 50 years. Among the influencing factors, textbooks have played a major role [15]. As a result, Japan has achieved high performance in education. According to Japan's curriculum, **Grade 7** textbooks introduce students to mathematical statistics through basic concepts such as data presentation, diagrams, and line graphs. In **Grade 8**, statistical topics include: *Frequency distribution tables*, *Frequency histograms and polygons*, *The mode*, *The mean*, and *The range*. These topics are continued in **Grade 9**. In **Grade 10**, additional content includes dependent and independent diagrams. These topics are placed at the end of the textbook and continued at the beginning of the **Grade 11** textbook. Statistical topics are thus sequentially and conceptually connected across grades. In **Grade 11**, mathematical statistics topics are presented at the beginning, while *Probability Theory* topics such as *Rolling a pair of dice* and *Experimental probability* are included at the end.

C. United States (California)

The United States also ranks high in international assessments such as PISA. In the US, each state has its own curriculum and textbooks. In the state of **California**, Grade 7 mathematics textbooks introduce *Probability Theory* with topics such as *Probability of Events* and *Making predictions using theoretical and experimental probability*. Elements of *Mathematical Statistics* appear in **Grades 8 and 9**. In **Grade 8**, prior to statistics, students study functions and their graphs, methods of representing functions, and generating functions from tabular data. Subsequently, as a continuation of these topics, the curriculum introduces: *Creating scatter plots*, *Identifying lines of best fit*, and *Writing equations for those lines* (as seen in Table 1). **Grade 9** continues with topics on functions and types of functions before introducing further statistical and probabilistic concepts. In **Grades 10 and 11**, the textbooks include content on *Probability Theory*. In **Grade 10**, students are introduced to *Theoretical and experimental probability*, *Identifying dependent and independent events*, *Calculating conditional probability*, and *Finding probabilities using two-way frequency tables* through different approaches.

D. Russia

Russia is also among the countries with a leading educational system. In Russian textbooks, elements of *Mathematical Statistics* are introduced starting from **Grade 7**. Initially, students are trained to solve problems related to numerical expressions. Then they are introduced to topics such as *Samples*, *Series of variants*, *Sample size*, *Arithmetic mean*, *Mode*, and *Median*. Understanding numerical expressions makes it easier to comprehend variant series and related problems. Statistical topics in these textbooks are explained in a very systematic manner and are enriched with practical problems. In **Grade 8**, these topics are continued with *Statistical characteristics*, *Frequency tables*, and *Introduction to the interval method*. In Russia, *Probability Theory* is introduced in **Grade 9**. Initially, students are introduced to *Statements and predicates*, *Quantifiers*, and *Operations with logical statements*

and predicates. Following this, probability topics are presented. In **Grades 10 and 11**, probability topics are further expanded. Additionally, in **Grade 11**, topics related to elements of *Combinatorics* are introduced.

4. Conclusion

In Singapore, the topics of *Probability Theory* are covered in the 8th and 10th grade textbooks. In these grades, particular attention is given to *Probability of Events*, including *Simple and Compound Events*. In 7th and 9th grades, statistical topics are presented, mainly focusing on the *collection of statistical data* and *calculating numerical characteristics*.

In Japan, topics on *Probability Theory* appear only in the 11th grade textbook. Compared to other countries, fewer instructional hours are allocated to this subject. In all other grades, the focus is primarily on *Statistics*.

According to the curriculum of the United States, more emphasis is placed on *Random Events*, *Theoretical and Experimental Probability*, and *Prediction based on probability*. In this country, *Probability Theory* is taught in connection with real-life situations, emphasizing practical applications.

In Russia, before introducing *Probability Theory*, students first study *Statements and Predicates*. Mastery of these concepts facilitates better understanding of *Random Events*. Afterwards, the curriculum introduces *Combinatorics* (permutations, arrangements, combinations), which supports effective learning of event probabilities and their calculations. In grades 10–11, students study *Axioms of Probability Theory*, *Random Variables*, and *Probability Distributions*.

As seen from the practices of the countries examined, topics on *Probability Theory* and *Statistics* in textbooks are logically connected with preceding topics. Moreover, the examples given are closely related to real-life situations, making the learning process more effective, engaging, and meaningful, thus facilitating better comprehension of the subject matter.

In addition, the following recommendations are proposed:

The content of the *Probability Theory* and *Mathematical Statistics* sections should be improved by prioritizing the development of students' skills in analyzing probabilistic situations and interpreting statistical data.

The didactic and methodological capabilities, as well as instructional and methodological resources for teaching probability and statistics, should be enhanced by aligning them with active learning methods.

The methodology for teaching *Probability Theory* and *Mathematical Statistics* should be updated by integrating approaches that ensure the harmony of theory and practice.

A criterion for assessing the level of students' ability to analyze probabilistic situations and interpret statistical data effectively should be developed.

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